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Addressing Challenging Mathematics Standards With At-Risk Learners: A Randomized Controlled Trial on the Effects of Fractions Intervention at Third Grade

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Abstract

The purposes of this study were to assess the effects of fractions intervention for students who are at risk for poor outcomes and to examine whether a component that combines self-regulated learning with growth-mindset instruction (SR-GM) provides added value for improving outcomes. At-risk students ($N = 84$) were randomly assigned to three conditions: fractions intervention, fractions intervention with embedded SR-GM, and a control group. Intervention was conducted three times per week for 35 min per session for 13 weeks. Multilevel models indicated both fractions intervention conditions produced strong effects, with no added value for SR-GM. Posttest fractions achievement gaps for both intervention conditions held steady, narrowed, or closed, whereas the control group's gaps remained sizeable or grew. Results suggest that intervention can address challenging mathematics standards for at-risk learners and that SR-GM instruction may not be necessary in the context of strong intervention.

Many students experience difficulty understanding and operating with fractions (Namkung et al., 2018; National Mathematics Advisory Panel [NMAP], 2008). This is problematic because competence with fractions predicts high school algebra and overall math achievement (Siegler et al., 2012). NMAP (2008) therefore assigned priority to improving fractions performance. It also recommended a strong instructional emphasis on understanding fraction magnitude, such as comparing, ordering, and placing fractions on number lines.

Although difficulty with fractions is pervasive, it is especially severe among students who develop mathematics deficits in the primary grades (Namkung et al., 2018). Fuchs and colleagues thus undertook a program of research to develop a fractions intervention focused on magnitude understanding for students who begin fourth grade with low math skill. As revealed in a series of randomized controlled trials (Fuchs et al., 2013, 2014; Fuchs, Malone, et al., 2016a; Fuchs, Schumacher, et al., 2016; Malone et al., in press, intervention with explicit, structured instructional strategies on fraction magnitude substantially narrows this population's fractions achievement gap at end of fourth grade.

Yet, few studies have examined whether intervention enhances the fractions performance of third graders who start the year with poor math skill (referred to as *at-risk students*

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in this article). This is despite that college- and career-ready standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) establish the expectation that third graders will develop understanding of fractions as numbers. With increased third-grade classroom focus on fractions, at-risk students who do not receive fractions intervention may be at increased risk for completing third grade with substantial performance deficits in this domain and for exacerbated mathematics difficulty in later grades.

At-risk students who do not receive fractions intervention may be at increased risk for completing third grade with substantial deficits in this domain and for exacerbated mathematics difficulty in later grades.

Therefore, the present study's first purpose was to assess the effects of fractions intervention on at-risk third graders. The second purpose was to assess whether an intervention component that combines self-regulated learning with growth-mindset instruction (SR-GM) provides added value over the same fractions intervention without SR-GM. (Note that an earlier iteration of this fractions intervention and SR-GM was tested; in the present study, we report effects of the subsequent iteration. In the present study, we did not compare iterations or isolate effects of the present intervention's design features other than SR-GM. To gain insight into effects of other components or design features, other component analysis studies are required.)

Effects of Fractions Intervention on At-Risk Third Graders

We identified four prior experimental studies testing the effects of fractions intervention with at-risk third graders. In locating prior studies, we defined *intervention* as instruction that is supplemental to the classroom program and delivered in small groups or individually. Perkins and Cullinan (1984) used a multiple-baseline design to assess effects of direct instruction with three third graders, one of whom was drawn from a low-performing class. The intervention's effect on fractions performance was demonstrated with all three children. However, the focus of instruction and outcomes was adding fractions with like denominators and part–whole understanding: representing fractions with circles, writing numerical fractions for circle representations, and adding fractions with like denominators. The focus on magnitude understanding was limited to identifying fractions as greater than, equal to, or less than 1.

Courey (2006) isolated the effect of a teacher language component when teaching visual representations of halves in the context of a 3-week word-problem intervention. Participants (50 of 51 identified with low math) were randomly assigned to a control group and two intervention conditions, one with and one without the language component. In this component, teachers conveyed the meaning of *half* as "one of two equal parts," and students practiced identifying numbers, word, or phrases that expressed the number to be halved or communicated finding two equal parts. This language component provided no added value over the same intervention without this component; students in both intervention groups outperformed the control group on procedural but not fraction understanding outcomes. Moreover, as with Perkins and Cullinan (1984), the major emphasis was part–whole, not fraction magnitude understanding.

Hunt (2014) randomly assigned 38 students to a core instruction control group or to receive core instruction plus explicit, conceptual intervention on ratio interpretation of rational numbers. Ratio interpretation of fractions relies more on part–whole understanding and equal sharing than on fraction magnitude, but the intervention included a smaller focus on magnitude. Among 19 students identified with initially low math skill, means favored intervention over control on converting fractions and mixed numbers as well as on fraction equivalency (standard deviations were not provided; moderation for initially low vs. typical math skill was not tested). In Ennis and Losinski's (2019) meta-analysis, they reported an effect size (ES) of 2.50 for this study.

By contrast, in the final third-grade intervention study, Wang et al. (2019) centered on fraction magnitude understanding, that is, teaching the conceptual bases for fraction magnitude and systematic cognitive strategies for reasoning through fraction magnitude when comparing and ordering fractions, placing fractions on number lines, finding equivalences, and solving word problems (WPs) that contextualize reasoning about fraction as number in everyday contexts.

Wang et al.'s (2019) general instructional approach is based on the idea that fraction magnitude understanding, generally and in the context of word-problem solving, transparently demands reasoning ability. It also taxes working memory: Magnitude comparisons require children to store and access information across a series of steps, which include finding equivalent fractions that need to be compared to benchmark fractions; fraction WPs require children to process text describing a series of quantities, which must be sequentially evaluated and iteratively considered to build a coherent problem-solving model. Also, these tasks involve language comprehension, because teachers use language to convey new ideas and procedures and because WPs are presented linguistically. In longitudinal studies predicting children's development of fraction and word-problem knowledge, roles have been demonstrated for reasoning (e.g., Fuchs, Malone, et al., 2016), working memory (e.g., Jordan et al., 2013), and language comprehension (e.g., Jordan et al., 2013).

Accordingly, the intervention's instructional approach is designed to compensate for limitations in these cognitive resources via explicit, structured cognitive strategy instruction, as follows. Tutors introduce new topics with worked examples by modeling efficient solution strategies using simple, direct language to explain and think aloud each step of strategies. Efficient solutions capture the essential ideas underpinning a problem type and lead to accurate solutions in as few steps as possible. Tutors fade worked examples as students gradually assume responsibility for applying and explaining strategies. Guided and independent practice is distributed, with cumulative review systematically woven through lessons and with interleaved problem sets requiring students to discriminate among problem types. Tutors provide corrective feedback for incorrect responses and incorrect student explanations.

Wang et al. (2019) randomly assigned 69 students to a control group and two intervention conditions, one with and one without SR-GM. Effects contrasting the fractions intervention condition without SR-GM against the control group were mixed. On fractions ordering, the effect was significant, with a large ES (1.29). Yet, on WPs, a significant main effect was moderated, such that positive effects were associated with stronger pretest word-problem skill. Further, on number line, a critical indicator of fraction magnitude understanding (Hamdan & Gunderson, 2017), and on the study's transfer measure, effects were not significant (ESs = 0.15 and 0.12).

Wang et al. (2019) thus revealed the need to strengthen the efficacy of the third-grade fraction magnitude intervention. Toward that end, we made five major changes. We slowed the pace for introducing new content to provide students time to develop deeper understanding and firmer mastery. We consolidated strategies to integrate magnitude understanding and strategy use across comparing, ordering, and number-line activities. We added instruction to highlight similarities and differences in the thought processes among the three magnitude activities. We relied on interleaved magnitude problem sets to provide systematic practice discriminating among problem types. And we reduced the number of word-problem types to provide more students time to develop understanding of the remaining two problem types. (See additional information in Supplemental Table 1 online.)

The first purpose of the present study was to assess the overall effect of this new iteration of the third-grade fraction magnitude intervention. Estimating this effect is important because, as reflected in the paucity and focus of prior studies, minimal attention has been allocated at third grade to fractions intervention and specifically to fraction magnitude intervention. Inadequate attention to fractions is likely due to supplemental intervention's dominant focus on remediation (Powell & Fuchs, 2015) or to teacher skepticism about the appropriateness of grade-level standards for students with disabilities (Edgerton et al., 2020).

Yet, standards reform establishes the expectation that all students, including those with disabilities, achieve college- and career-ready standards (Edgerton et al., 2020). By assessing the effects of the next-iteration intervention, the present study addresses a pressing question in the context of standards reform and the policy of access to the general education curriculum: Can at-risk students who receive intervention succeed with challenging mathematics standards?

We operationalized challenging standards via fraction magnitude, including WPs. By *succeed*, we mean at-risk intervention students (a) significantly outperforming at-risk control group students with strong ESs and (b) completing intervention with substantially narrowed achievement gaps with respect to not-at-risk classmates. We hypothesized that this next-iteration intervention, with its slower pace, integrated focus across magnitude activities, interleaved practice, and more intense focus on two word-problem types for contextualizing reasoning about fractions as number would permit at-risk third graders to succeed.

Added Value of Self-Regulation With Growth-Mindset Instruction

Although Wang et al. (2019) revealed the need to strengthen the fractions intervention, findings suggested promise for a focus on SR-GM. On all measures except number line, the contrast between the fractions intervention with SR-GM versus the control group was significant, and ESs were moderate to strong (0.55–1.76; ESs for the contrast between the fractions intervention without SR-GM vs. control were lower: 0.15–1.29).

SR-GM integrates active self-monitoring and goal setting with a related construct: growth mindset, in which individuals believe intelligence can change (cf. Dommett et al., 2013). Research suggests that growth mindset predicts achievement (e.g., Blackwell et al., 2007). Some studies have tested the effects of growth-mindset instruction on academic achievement, with mixed findings. For example, Yeager et al. (2019) conducted a randomized controlled trial examining effects of one growth-mindset session on ninth graders with initial grade point average (GPA) below the school's median. Results indicated stronger final mathematics GPA and increased enrollment in advanced math courses. By contrast, Dommett et al. (2013), who randomly assigned schools to receive brain plasticity workshops or control, found no significant effects on 11- to 12-year-olds' math outcomes. We identified no prior studies at third grade.

With active self-monitoring and goal setting, students rely on progress monitoring to formatively evaluate progress and set goals. Monitoring progress against a standard is thought to help students adjust skills and strategies to improve learning (Graham & Harris, 1997). Goal setting is thought to mobilize and sustain effort to achieve objectives (Cervone, 1993).

One widely researched and relevant self-regulated-learning (SR) approach for at-risk intervention is Self-Regulated Strategy Development (SRSD; Harris et al., 2015). It embeds self-monitoring and goal-setting instruction within academic cognitive strategy instruction. The What Works Clearinghouse (WWC; 2017) conducted a systematic review of SRSD. Its conclusion was that SRSD has potentially positive effects on the writing achievement of students with learning disabilities. Graham and Harris (1989) isolated effects of SR (self-assessment and goal setting without growth-mindset instruction) within SRSD by randomly assigning students with learning disabilities to receive SRSD's writing cognitive strategies with versus without SR. Results indicated that SR did not augment the effects of academic strategy instruction on writing performance. In fact, mean differences favored the condition without SR.

In mathematics, we located three single-case-design SRSD studies involving students with or at-risk for learning disabilities. Case et al. (1992) demonstrated improvement on simple WPs involving addition and subtraction among four fifth and sixth graders. Cuenca-Carlino et al. (2016) showed a functional relation between SRSD and four middle school students' multistep equation solving within WPs. Most pertinently, Losinski et al. (2019) provided evidence of a functional relation between SRSD and fractions calculations skill for 15 of 16 at-risk participants.

In math, however, we identified no study, except Wang et al. (2019), that isolated SR's effect in the context of SRSD or any other approach to math intervention. We did identify relevant studies conducted at the classroom level. For example, De Corte et al. (2000) examined whether classrooms might be designed to foster SR to support mathematical problem solving. Yet, effects cannot be attributed to SR because those design experiments incorporated multiple innovative principles, with varying levels of experimental control. In a third-grade classroom study with experimental control, Fuchs, Fuchs, et al. (2003) randomly assigned classrooms to word-problem instruction with versus without SR. ESs between the two conditions showed promise, with ESs of 0.24 to 0.58 for at-risk math students.

In the absence of prior intervention studies isolating effects of positive growth mindset or self-assessment and goal setting on at-risk students' math achievement, along with promising ESs for a classroom study isolating effects of self-assessment and goal setting on at-risk learners' math performance, Wang et al. (2019) extended the framework for SR-GM by combining flexible growth mindset with self-assessment and goal setting. The rationale was synergy: Growth mindset may encourage at-risk students to persevere through challenging fractions content and set ambitious expectations for themselves, even as tracking improvement as a function of hard work may build flexible growth mindset.

The general instructional approach in Wang et al. (2019)'s SR-GM component is consistent with the explicit, structured approach described for the fractions intervention. Further, as in SRSD, we embedded this innovative SR-GM component into fractions cognitive strategies intervention because SR-GM is unlikely to enhance academic competence of at-risk learners if it occurs without concurrent academic skill building (e.g., Catts & Kamhi, 2017; Melby-Lervag & Hulme, 2013). Further, embedding SR-GM provides tutors opportunities throughout fractions instruction to invoke the principles taught in SR-GM lessons.

Thus, the present study's second purpose was to isolate SR-GM's added value over a fractions intervention without SR-GM in the context of a revised, hopefully strengthened fractions intervention. Just as we attempted to strengthen the fractions intervention, we tried to enrich SR-GM in two ways. We infused existing SR-GM with scenarios, conveyed via comics, depicting similarly aged students with similar struggles engaging in the taught SR-GM processes. The hope was to help learners understand the relevance and value of the ideas conveyed in the SR-GM lessons for their own fractions learning. Support for this approach is found in social learning theory (Bandura, 1986) and prior studies conveying information to students via comics (Obare et al., 2013). For example, Mitchell and Milan (1983) used comic strip models to improve classroom behavior in young children. We also added a focus on students checking their own work for sources of errors and misunderstandings, and we encouraged them to use this information to adjust plans and select practice items to reach goals. Following Wang et al. (2019), and the potential of these changes to strengthen SR-GM, we hypothesized added value for revised fractions intervention with revised SR-GM over the revised fractions intervention without SR-GM.

Method

The present randomized controlled trial was designed with three conditions: fractions intervention on improving fraction understanding and WPs (FRAX), the same intervention with SR-GM (FRAX+SR-GM), and a business-as-usual control group (regular classroom instruction, with some students receiving the school's intervention). We controlled for instructional time across the two intervention conditions as described later. The Vanderbilt University Institutional Review Board governed treatment of human subjects. Participating teachers and parents of participating children provided consent. Students provided assent.

Participants

At-risk participants were third-grade students at risk for or with identified learning disabilities from 29 classrooms in eight schools in a metropolitan school district. We conducted whole-class screening at the start of the school year to identify students who met either or both of two low-math criteria, as in Wang et al. (2019): (a) scoring below the 22nd percentile on a broad-based math measure (Wide Range Achievement Test–4 [WRAT]; Wilkinson & Robertson, 2006), which involves whole-number and fractions numeracy and calculations and (b) scoring below the 31st percentile on WRAT and below 3 on the Minuends to 18 subtest of the Second-Grade Calculations Battery (Fuchs, Hamlett, et al., 2003). Due to WRAT's thin behavior sampling at each grade, scoring one additional item correct moves third graders from the 22nd to the 31st percentile. To avoid missing some students in need of intervention, the second criterion involved a measure with thick sampling of single-digit subtraction, which is more difficult than addition (Baroody, 1984; Fuson, 1984) but typically consolidated by third grade (Nelson & Powell, 2018).

Of 406 screened students, 151 met one or both criteria. We excluded 45 students: 10 whose teachers identified them as having limited English proficiency (to avoid false positive identification of risk), three with an autism diagnosis or an intensive behavior plan, 19 scoring less than the 9th percentile on both subtests of the Wechsler Abbreviated Scales of Intelligence (WASI; Wechsler, 2011; we made these exclusions because intervention was designed to address the needs of students at risk for or with learning disabilities), and 13 whose schedules precluded participation.

We randomly assigned 90 of the remaining 106 at the individual level to three conditions (i.e., students in the same classroom participated in different conditions, and students in the same dyad were not necessarily from the same classroom). There were 30 students per condition: business-as-usual control group, FRAX, and the same intervention with embedded SR-GM (FRAX+SR-GM). Prior to the study's end, four FRAX, one FRAX+SR-GM, and one control student moved outside the county. Complete data were thus available for 26 FRAX-condition students, 29 FRAX+SR-GM-condition students, and 29 control students. Students did not differ by condition on demographics or screening variables (see Tables 1 and 2).

Also, to judge the severity of at-risk students' pre- and posttest achievement gaps on fractions, we also followed a sample of not-at-risk classmates, randomly sampled from those meeting neither of the low-math study entry criteria. During the study, 15 moved to schools outside the county. This left 194 not-at-risk classmates, who were approximately 2.5 standard deviations above at-risk classmates on WRAT-4. Not-at-risk classmates completed a subset of the study's fraction measures. This subset of measures is described in the online supplement, which also provides pre- and posttest means and standard deviations and ESs indicating the magnitude of achievement gaps on these measures by risk and intervention status (see Supplemental Table 2 online).

Screening Measures

With WRAT-4 Math Computation subtest (Wilkinson & Robertson, 2006), students complete an oral section with 10 whole-number numeracy items and a written section with 40 numeracy, conceptual, and procedural whole- and rational-number problems of increasing difficulty (median reliability at 5–12 years = .94). With Second-Grade Calculations Battery Minuends to 18 (Fuchs, Hamlett, et al., 2003), students have 1 min to complete 25 problems (α = .89). WASI (Wechsler, 2011), a two-subtest measure of general cognitive ability,

Table 1. Student Demographics by Study Condition.

	FRAX Condition (*n* = 26)		FRAX+SR-GM Condition (*n* = 29)		Control Condition (*n* = 29)	
Variable	*n*	%	*n*	%	*n*	%
Male	11	42.3	14	48.3	16	55.2
Race-ethnicity						
African American	16	61.5	18	62.1	13	44.8
Caucasian	2	7.7	2	6.9	3	10.3
Hispanic	5	19.2	8	27.6	12	41.4
Other[a]	3	11.5	1	3.4	1	3.4
Subsidized lunch	15	57.7	16	55.2	15	51.7
School-identified disability						
Learning disability	1	3.8	2	6.9	1	3.4
Learning disability and behavior disorder	—	—	—	—	—	—
Speech-language delay	—	—	—	—	3	10.3
Other	—	—	—	—	2	6.9
English language learner	5	19.2	7	24.1	9	31.0

Note. FRAX = fractions intervention without the self-regulated component; FRAX+SR-GM = fractions intervention with self-regulated learning with growth-mindset instruction.
[a]There were no Asian American participants.

Table 2. Pretest Scores by Study Condition.

	FRAX condition (*n* = 26)		FRAX+SR-GM condition (*n* = 29)		Control condition (*n* = 29)	
Screening measure	*M*	*SD*	*M*	*SD*	*M*	*SD*
WRAT-4 Math Computation	21.81	1.44	21.45	1.23	21.38	1.15
WASI Matrix Reasoning	8.31	2.41	8.72	3.95	8.38	2.56
WASI Vocabulary	20.23	3.98	19.76	4.58	19.31	4.63

Note. FRAX = fractions intervention without the self-regulated component; FRAX+SR-GM = fractions intervention with self-regulated learning with growth-mindset instruction; WRAT-4 = Wide Range Achievement Test (Wilkinson & Robertson, 2006); WASI = Wechsler Abbreviated Scales of Intelligence (Wechsler, 2011).

includes Vocabulary and Matrix Reasoning (split-half reliability > .92). Vocabulary assesses expressive vocabulary, verbal knowledge, memory, learning ability, and crystallized and general intelligence. Matrix Reasoning measures nonverbal fluid reasoning and general intelligence.

Fraction Outcome Measures

Four measures (multiplication, ordering, number line, addition and subtraction) were researcher designed and assessed acquisition of skills taught in intervention. The fifth, also researcher designed, assessed a mix of acquisition and transfer WPs, none of which was used during intervention. The sixth, the study's main transfer measure of generalized fraction knowledge, was not researcher designed.

With Fraction Battery–Revised Single-Digit Multiplication (Malone & Fuchs, 2017), students have 5 min to answer 30 problems (factors 1–10) shown horizontally (α = .92). We included multiplication as a fraction outcome because it is foundational for identifying equivalent fractions and is addressed in the intervention conditions.

Fraction Battery–Revised Ordering (Malone & Fuchs, 2017) assesses magnitude understanding with 12 items. (We relied on ordering instead of comparing because students can cross-multiply to solve comparing but not ordering problems.) Each ordering problem shows three fractions to be ordered from least to greatest. Two items have fractions with the same numerator, one has fractions with the same denominator, and the remaining nine include 1/2 as one of the three fractions. The maximum score is 12 (α = .82). To control for pretest performance, while avoiding floor effects due to limited skill at start of third grade, we used Fraction Battery–Revised Comparing at pretest. With Comparing (six items), students place a greater-than, less-than, or equal sign between two fractions. Two items have the same numerator, one has 1/2 and a fraction less than 1/2, one has 1/2 and a fraction equivalent to 1/2, one can be solved by rewriting one fraction with an equivalency to make the same denominator or numerator as the other fraction, and one has a fraction equal to 1 and a fraction less than 1. The maximum score is 6 (α = .82).

Fraction Battery–Revised Number Line (Malone & Fuchs, 2017) assesses fraction magnitude by having students place fractions on a 0-to-1 paper number line. Testers demonstrate the mechanics of using tick marks to place fractions on number lines. Students then complete six items. With each, students place two fractions on the same number line labeled with endpoints (0 and 1). For each item, students earn 1 point for placing each fraction correctly above or below 1/2 and 1 point for placing the two fractions in correct order, regardless of whether the fraction is on the correct side of 1/2. The maximum score is 18 (α = .86).

Fraction Battery–Revised Word Problems (Malone & Fuchs, 2017) includes 18 acquisition and transfer WPs, none used during intervention. Six are compare WPs; 12 are change WPs. (The pretest includes 10 items to limit fatigue given students' limited skill.) Compare WPs require students to evaluate fraction magnitudes in a comparison narrative (e.g., "In art class, Maria used 5/12 of a bottle of blue paint and 3/4 of a bottle of red paint. What paint color did she use more of?"). Some problems include irrelevant numerical information or an additional fraction requiring students to order. Change problems require students to solve for a missing start, change, or end amount in a cause-effect narrative (e.g., "Kavonte had 5/6 of a bottle of water. He drank 2/6 of the bottle of water. How much water does he have now?"; 5/6 is the start amount; 2/6 is the change amount; the end amount is missing). Some problems include irrelevant numerical information. Testers read items aloud while students follow on paper. Students can ask for one rereading. For each problem, students earn 1 point for the correct numerical answer and 1 point for the correct label (of a bottle of water) or 0.5 points for partial labeling (bottle of water). Awarding credit for labels helps index students' understanding of problem models and fractions. The maximum score on the 18-item test is 36 (α = .89).

Fraction Battery–Revised Addition and Subtraction (Malone & Fuchs, 2017) includes 14 fraction addition and subtraction problems with like (seven items) and unlike (seven items) denominators. To solve problems with unlike denominators, students rewrite "1/2" as an equivalent fraction. Four items that include 1/2 as a fraction are subtraction; four are addition. Students earn 1 point for each correct answer. The maximum score is 14 (α = .90).

The study's transfer measure, indexing generalized fraction knowledge, comprises 13 released items from the National Assessment of Educational Progress (NAEP): a subset of easy, medium, or hard fraction items from the fourth-grade assessment and a subset of easy items from the eighth-grade assessment. See Supplemental Figure 1 online. Items tap part–whole and magnitude understanding via problem types or response formats not addressed in intervention (except one subtraction-with-like-denominators item). Testers read problems aloud. The maximum score is 13 (α = .63).

Fractions Intervention Common to Both Intervention Conditions

In the FRAX and the FRAX+SR-GM conditions, teachers and students refer to the intervention as Super Solvers–Third Grade–Revised (Fuchs, Malone et al., 2019). For more information on lesson activities, see Supplemental Figure 2 (for FRAX content) and Supplemental Figure 3 (for SR-GM content) online. To replicate or implement FRAX or the SR-GM component or to obtain a copy of the Super Solvers–Third Grade manual, with all lessons and materials, contact the first author or go to https://frg.vkcsites.org/.

Intervention includes three 35-min sessions per week for 13 weeks. In the present study, it was delivered to pairs of students. The FRAX+SR-GM condition differs from the FRAX condition in that SR-GM students receive lessons on SR-GM lessons, principles that are systematically invoked during the FRAX content. The SR-GM content takes 4 to 9 min per session. In the present study, instructional time between the two intervention conditions was held constant by providing FRAX students more time for independent practice and, beginning with Lesson 22, extra WPs. See the introduction for an overview of the intervention's structured, systematic instructional design guiding FRAX and SR-GM lessons.

The focus in FRAX is fraction magnitude, with comparing, ordering, and placing fractions on number lines, all which involve proficiency with fraction equivalencies, as well as WPs to contextualize fractions as number in everyday contexts. WP instruction, which is schema based (Fuchs, Malone, et al., 2016; Wang et al., 2019), focuses on compare fraction WPs and change fraction WPs (see Measures for definitions and examples of WP types). Each 35-min lesson includes up to five activities: Multi-Minute (1–2 min), Problem Quest (7–12 min), Fraction Action (10–18 min), Fraction Flash (2–3 min), and Power Practice (5–7 min).

During Multi-Minute (Weeks 1–13), students practice whole-number multiplication and learn strategies for solving basic facts (1s through 10s). This relies heavily on skip-counting practice with the assistance of a skip-counting mat. In Week 7, tutors introduce Multi-Minute Flash, which lasts through Lesson 39. With Multi-Minute Flash, tutors present multiplication facts, and students alternate with their partner to provide as many correct responses as possible in 1 min; when an error occurs, the tutor requires the student to use skip-counting to derive the correct response before the next card is revealed. To discourage careless responding, the timer continues to run. Pairs try to beat their previous session's score.

Fraction Action (Weeks 1–13) addresses fraction magnitude. In Weeks 1 through 8, students extend prior part–whole and equal-sharing understanding. Foundational lessons have a strong emphasis on fraction vocabulary. Activities include comparing, ordering, and placing fractions on number lines while finding equivalencies. Throughout all lessons, fraction tiles, fraction circles, and number lines are used to introduce and review concepts throughout the program. Instruction is supported via strategies presented on the *compare* card, and guided problem solving for the compare, order, and number-line fraction magnitude problem types. The *compare* card is faded as quickly as possible.

Students learn same-denominator and same-numerator conceptual comparing strategies focused on what the numerator and denominator mean. Then, comparing fractions with different denominators or different numerators begins. Tutors teach strategies for identifying fractions equal to 1 (when numerator and the denominator are the same) and fractions equal to 1/2 (double the numerator should equal the denominator or the numerator is half the denominator). Then benchmarking instruction starts, with assistance of problem-solving strategies on the *compare* card. In Week 8, students learn to place two fractions less or greater than 1/2 on a 0-to-1 number line; this requires finding an equivalent fraction. Tutors next introduce ordering. Once all activities have been introduced, tutors lead discussions about similarities and differences in problem types to emphasize that the same thinking and strategies apply across fraction magnitude activities.

Fraction Flash (Weeks 1–13) is designed to build flexibility and speed with fraction magnitude component skills. The structure of this activity parallels the structure used in Multi-Minute. Depending on week, this includes stating which of two fractions is bigger; saying if fractions = 1/2, = 1, or neither; and pointing to "1/2" on a number line; identifying if the fraction is less than, equal to, or greater than 1/2; and pointing to which side of 1/2 the fraction goes.

Problem Quest (Weeks 4–13) addresses WP instruction, beginning in Week 4. Relying on schema-based instruction (Fuchs, Malone, et al., 2016), tutors teach students to categorize WPs as belonging to a problem type based on its underlying mathematical structure and using the "RUN" attack strategy: read the problem, underline the question, and name the problem type.

Tutors introduce each WP type (compare WPs, change WPs) with an intact story while explaining and demonstrating the WP type's central mathematical event with fraction tiles. Next, they present the same story in the form of a WP, with an unknown and a question. Then, students learn a systematic strategy for processing and solving that problem type. To execute the strategy, students initially use a *help* card, which is faded as quickly as possible.

Compare WPs are taught first. In Lesson 20, ordering WPs are taught as a subtype of compare WPs. In Lesson 21, compare WPs with irrelevant information are introduced. Strategic introduction of compare WP variations encourages students to distinguish ordering WPs from compare WPs with irrelevant fractions in the story. In Week 5, tutors introduce increase and decrease change WPs with the end amount missing, using whole numbers and then fractions. In Week 8, tutors introduce problems with irrelevant information while encouraging students to distinguish between compare and change WP types. In Week 9, change WPs with change amount missing are introduced, first using whole numbers, then fractions; in Week 10, start-amount-missing problems are introduced, first using whole numbers and then fractions.

In Power Practice (Weeks 1–13), the final activity in each lesson, students independently complete interleaved problem sets (with mixed problem types) of previously taught compare, ordering, and number-line problems. Starting in Week 4, practice also includes one WP. By Lesson 22 (when most content is introduced), independent practice (12–14 problems per session) presents each of the 20 problem types addressed in intervention at least every two lessons. Problem types within a topic (i.e., within fraction magnitude or WPs) are randomly ordered.

Intervention also incorporates a motivational system focused on on-task behavior: "Listen, try your best, and be respectful." Tutors set a timer to beep at three unpredictable intervals each session. Student who are on task at the beep earn a "dollar" for their "bank account." Students also earn dollars for on-task behavior during transitions between classrooms and during intervention. In Power Practice, they earn bonus dollars for accurately completing problems. At each lesson's end, students purchase a prize from the Super Store or save money for a more highly valued purchase. Also, starting Week 3, students in both conditions complete a fractions curriculum-based measurement (CBM) progress-monitoring probe, called Super Challenge, every 2 weeks. Each includes 20 problems representing the fraction magnitude problem types.

The SR-GM Component

With the SR-GM component, students receive the same FRAX intervention but with the SR-GM component integrating instruction on growth mindset with self-assessment and goal setting. This includes feedback and goal-directed discussion after each Super Challenge CBM and a *Brain Boost* adventure with discussion on growth mindset at the start of each lesson (see Supplemental Figure 4 for a sample *Brain Boost* episode). (Although CBMs were conducted in both intervention conditions, only SR-GM students graphed and discussed progress, set goals, and adjusted plans to reach self-set goals. In FRAX without SR-GM, tutors scored assessments but without guiding student reflection on progress.)

The *Brain Boost Adventures* comics address the key SR-GM concepts already described. We operationalized the idea of growth mindset as "brain power can grow," referring lessons to supporting research. During Weeks 1 through 3, *Brain Boost Adventures* comics focuses on teaching students about "brain power," its malleability, how to train the brain like an athlete, how mistakes can help the brain grow, and tracking progress and goal setting. Students are explicitly taught how to graph and interpret their graphs (see Supplemental Figure 5 for a sample graph) and how to set goals to beat their highest score.

In Week 4, discussion extends discussion to learning from mistakes. Students follow *Brain Boost Adventures* to examine and discuss careless mistakes and apply this thinking in their first Super Challenge. In Lesson 10, students review their Lesson 9 CBM to identify mistakes. Tutors prompt students to think, "Why did I get this type of problem wrong?" (e.g., forgetting a strategy vs. making a careless mistake) and "What can I do to get it right?" In Week 5, *Brain Boost Adventures* encourage students to use fractions in everyday life, to persist in learning fractions, and to think about why their CBM scores increase, decrease, or stay the same. Week 6 *Brain Boost Adventures* teach how to set SMART (specific, measurable, achievable, realistic, time-bound) goals. In Weeks 7 through 13, *Brain Boost Adventures* emphasize working hard through challenges, prioritizing goals, adjusting plans to reach goals, and identifying strengths and weaknesses using CBM scores.

Tutor Training and Fidelity of Implementation

Ten tutors were research grant employees (three were licensed teachers). All had a bachelor's degree; two also had a master's. Each was responsible for two to four groups, distributed across the FRAX and FRAX+SR-GM conditions. To avoid contamination across conditions, we color coded materials, conducted periodic live observations, and monitored fidelity of implementation (FOI) audiotapes. Tutors also attended weekly meetings to receive condition-specific training for upcoming sessions, engage in problem solving, and receive feedback.

To quantify FOI, we audio recorded sessions. Of the 1,131 sessions, 20% were randomly sampled to ensure comparable representation of conditions, tutors, and lessons. Research assistants (RAs) listened to each recording while checking each essential point adhered to in the intervention protocol. For the FRAX, the mean percentage of points addressed was 92.05 (SD = 8.53) in the FRAX condition and 91.83 (SD = 6.15) in the FRAX+SR-GM condition. For the SR-GM component, the percentage of points addressed was 96.58 (SD = 10.28). Two RAs recoded 20% of sessions. Percentage agreement was 95% to 98%; a within-tutor paired t test indicated no significant difference for the FRAX component between conditions (p = .74).

Mathematics Instructional Time: Intervention Versus Control

Near the study's end, the 23 classroom teachers completed a survey on instructional time and practices. They reported that math instruction occurred in 80- to 90-min math blocks 5 days per week. The study's intervention (35 min three times per week) occurred during part of classroom math instruction or the school's intervention period. Students across the three study conditions received similar minutes of math instruction, including classroom instructional and supplemental intervention provided by the study or school: 284.35 min (SD = 103.41) in FRAX, 290.91 min (SD = 98.66) in FRAX+SR-GM, and 320.92 min (SD = 109.76) in control.

Fractions Instruction: Intervention Versus Control

As part of the survey, classroom teachers also provided information about the schools' fraction instruction. Of 23 teachers, 19 reported that fraction instruction was based largely on state standards. Four reported using a combination of the standards and the district's mathematics program (*GO Math!*; Houghton Mifflin Harcourt, 2015). See Supplemental Table 3 online for teacher responses describing the control group's fraction instruction, as contrasted to the researcher-provided fractions intervention.

Four main distinctions between the control group versus the two intervention conditions emerged. The control group focused mainly on part–whole understanding; the intervention conditions emphasized fraction magnitude. Second, to help students understand fraction magnitude, teachers relied primarily on number lines and picture drawing; the intervention conditions, although focusing heavily on number lines, also emphasized comparison with benchmark fractions with the meaning of the numerator and denominator, with no attention to picture drawing. Third, the control group did not restrict the range of fractions; the interventions conditions limited the pool of denominators to 1 to 10 and 12. Fourth, control-group WP instruction focused more on operational procedures and picture drawing; the intervention conditions focused more on identifying WPs as belonging to WP types to represent the structure of WPs, without any picture drawing.

Procedure

WRAT, Minuends to 18, and NAEP were completed in one 45-min whole-class session (late August to early September). WASI Vocabulary and Matrix Reasoning were administered individually in one 60-min session (mid-September to early October). Multiplication, Comparing Fractions, Fraction Addition and Subtraction, Fraction Number Line, and Fraction Word Problems–Pretest were administered in two small-group 45-min sessions (mid-September to early October). Intervention occurred for 13 weeks, three times per week for 35 min per session (October to early February). In late February to early March, posttest NAEP and Ordering Fractions (six items) were readministered in a whole-class session. Multiplication, Ordering Fractions (other six items), Fraction Number Line, Fraction Addition and Subtraction, and Fraction Word Problems were administered in two small-group sessions. Teachers completed instructional surveys in March.

Testers were RAs who received training and passed fidelity checks on testing procedures before administering tests. Two independent RAs scored and entered data. Scoring discrepancies were resolved. Test sessions were audiotaped; 20% of tapes were randomly selected, stratifying by tester, for accuracy checks by an independent scorer. Agreement on test administration accuracy was 98%. RAs were blind to study conditions when administering and scoring tests.

Data Analysis and Results

Table 3 shows pretest and posttest means by intervention condition (there were no missing data). Tests of baseline equivalence identified no significant differences among conditions on any pretest fraction measure except multiplication ($p < .05$). Preliminary tests indicated that pretest performance did not moderate intervention effects on any fraction outcome. Multilevel analyses were conducted with Mplus 8.2 (Muthén & Muthén, 1998–2018). Other preliminary analyses evaluated the nested structure of the data: a cross-classified, partially nested design in which nesting occurred at the school and classroom levels for all study conditions and at the intervention-dyad level only for the two intervention conditions. A three-level model with cross-classification of dyad and classrooms, both nested in schools, did not converge. So we used an indirect strategy to estimate the proportion of variance in each fraction outcome measure due to schools, classrooms, and intervention dyads: first regressing observations on school dummy codes and then modeling student data as nested in a cross-classification of classrooms and dyads using fixed effects, controlling for schools using dummy codes. The variance components from this pair of models were used to compute intraclass correlations (ICCs, i.e., the proportion of total variance in the specified outcome attributable to the specified level; see

Table 3. Pre- and Posttest Means and Standard Deviations.

	FRAX (n = 26)			FRAX+SR-GM (n = 29)			Control (n = 29)		
	Pretest	Posttest		Pretest	Posttest		Pretest	Posttest	
Measure	*M* (*SD*)	*M* (*SD*)	M_{adj}	*M* (*SD*)	*M* (*SD*)	M_{adj}	*M* (*SD*)	*M* (*SD*)	M_{adj}
Fraction Battery–Revised									
Number Line	3.27 (2.82)	8.77 (3.56)	8.78	3.83 (2.71)	9.17 (3.63)	9.06	2.83 (2.69)	5.14 (3.43)	5.24
Word Problems	1.94 (1.31)	10.48 (5.78)	10.16	1.78 (1.13)	10.40 (6.49)	10.35	1.53 (1.10)	4.76 (5.14)	5.10
Single-Digit Multiplication	8.81 (6.44)	19.42 (5.69)	18.11	6.69 (4.19)	18.86 (5.55)	18.66	3.69 (3.33)	10.76 (6.42)	12.11
Ordering[a]	0.38 (1.13)	5.27 (3.01)	5.26	0.21 (0.82)	5.38 (3.20)	5.39	0.24 (0.79)	2.24 (1.70)	2.25
Addition and Subtraction	0.04 (0.20)	5.31 (3.89)	5.35	0.00 (0.00)	4.66 (3.21)	4.74	0.17 (0.76)	1.72 (3.10)	1.60
NAEP	2.11 (1.33)	6.07 (1.89)	6.10	2.66 (1.21)	6.03 (2.64)	6.01	2.33 (1.63)	3.98 (2.21)	3.99

Note. FRAX = intervention without the self-regulated component; FRAX+SR-GM = intervention with self-regulated learning with growth-mindset instruction. NAEP = 13 released fraction items from the National Assessment of Educational Progress; M_{adj} = adjusted mean (i.e., posttest with pretest as a covariate).
[a]For Ordering, pretest is comparing fractions with the Fraction Battery–Revised Comparing (Malone & Fuchs, 2017).

Supplemental Table 4 online). ICCs were large enough to justify retaining school, classroom, and dyad in analyses.

We used the Roberts and Roberts (2005) method (described in Bauer et al., 2009) to model nesting for intervention conditions but not for the control condition. ICC analyses were modified accordingly; we obtained ICC results separately for each condition but sharing a common Level 1 residual variance. Next, we used Bayes estimation in Mplus to conduct regression models to test two orthogonal contrasts of interest: intervention (combined) versus control, and FRAX versus FRAX+SR-GM. Accordingly, the final model equation was

$$\begin{aligned} y_{ijk} &= \gamma_{00} + \sum_{m=1}^{7} \gamma_{0m} d_m + u_{0j} \\ &+ \left(\gamma_{10} + u_{1j} + u_{1k}\right) c_{1i} \\ &+ \left(\gamma_{20} + u_{2j} + u_{2k}\right) c_{2i} \\ &+ \gamma_{40} y_{0ijk} + e_{ijk} \end{aligned},$$

where y is a generic outcome, y_0 is pretest, c is dummy code for condition (00 = control; 10 = FRAX; 01 = FRAX+SR-GM), d is dummy code for school, i denotes individual student, j denotes classroom, k denotes dyad, and m is an index used for summing dummy code effects for schools. For FRAX+SR-GM versus FRAX, the difference was $\gamma_{20} - \gamma_{10}$. For average intervention (combined) versus control, the difference was $[(2\gamma_{00} + \gamma_{10} + \gamma_{20}) / 2] - \gamma_{00}$. Analyses used the ICC code as a basis, with pretest scores as covariates.

Results of the Bayes estimation are shown in Table 4, with credible intervals (CrI), rather than p values. CrIs that do not include zero indicate a significant effect. (With Bayesian estimation, a 95% CrI has a 95% probability of containing the parameter. Note that accounting for multiple comparisons is not necessary with Bayesian analysis because it is more conservative than frequentist analysis [Gelman et al., 2012]. Also, the tests for different dependent measures are independent, and only two hypothesis tests were conducted for each outcome.)

In line with the first study hypothesis, FRAX (combined across conditions) produced

Table 4. Results of Bayesian Estimates With Credible Intervals.

Contrast	Mean difference	95% credible interval	Significant	Condition with higher value
NAEP				
SR-GM vs. FRAX	0.173	[–1.570, 2.215]		
Intervention vs. control	2.208	[1.281, 3.207]	*	Intervention
Single-Digit Multiplication				
SR-GM vs. FRAX	0.355	[–3.990, 5.524]		
Intervention vs. control	5.901	[3.033, 8.251]	*	Intervention
Word Problems				
SR-GM vs. FRAX	0.633	[–3.125, 5.334]		
Intervention vs. control	5.163	[2.724, 7.493]	*	Intervention
Ordering[a]				
SR-GM vs. FRAX	0.316	[–2.167, 3.187]		
Intervention vs. control	3.182	[1.684, 4.841]	*	Intervention
Addition and Subtraction				
SR-GM vs. FRAX	–0.592	[–3.401, 2.936]		
Intervention vs. control	3.357	[1.572, 5.026]	*	Intervention
Number Line				
SR-GM vs. FRAX	0.390	[–2.656, 4.311]		
Intervention vs. control	3.426	[1.312, 5.104]	*	Intervention

Note. For contrasts, *intervention* refers to combined intervention conditions across conditions (without and with the self-regulated learning component). Number Line, Word Problems, Single-Digit Multiplication, Ordering, and Addition and Subtraction are from the Fraction Battery–Revised (Malone & Fuchs, 2017). NAEP = 13 released fraction items from the National Assessment of Educational Progress; SR-GM = fractions intervention with self-regulated learning with growth-mindset instruction; FRAX = fractions intervention without the self-regulated component.
[a]For Ordering, pretest is comparing fractions with the Fraction Battery–Revised Comparing (Malone & Fuchs, 2017).

higher scores than the control condition on each of the six study outcome measures. Yet, contrary to the second hypothesis, there was no significant difference between the two intervention conditions. ESs (Hedges *g*), calculated from adjusted posttest means, are shown in Table 5. (We provide ESs for the contrast between each intervention condition against control for reader edification, although those separate tests were not analyzed.) See Supplemental Table 2 for pre- and posttest achievement gaps with respect to not-at-risk classmates.

Discussion

The first purpose of this study was to assess the effects of fractions intervention on at-risk third graders. Wang et al. (2019) tested a previous iteration of the intervention, which revealed the need for further intervention development. In the present study, we tested the effects of the new iteration, designed in multiple ways to promote deeper understanding and firmer mastery. The second purpose was to examine whether an SR-GM component, which demonstrated promise in Wang et al. and was further extended in the present study, provides added value for improving student outcomes in the context of the next-iteration fractions intervention.

Effects of Fractions Intervention on At-Risk Third Graders' Fractions Performance

Results indicated that the fractions outcomes of at-risk students who received the next-iteration intervention were statistically significantly stronger than the outcomes of comparable at-risk students in the control group. ESs were strong: 1.06 on Single-Digit Multiplication, 1.03 on Number Line, 1.13 on Ordering, 0.88 on Word Problems, 1.00 on Addition and Subtraction, and 1.29 on NAEP for the combined intervention

Table 5. Effect Sizes.

Measure	Int vs. C	FRAX vs. C	FRAX+ SR-GM vs. C	FRAX+ SR-GM vs. Base
Fraction Battery–Revised				
Number Line	**1.03**	1.00	1.07	**0.08**
Word Problems	**0.88**	0.92	0.88	**0.03**
Single-Digit Multiplication	**1.06**	0.97	1.08	**0.10**
Ordering[a]	**1.13**	1.23	1.21	**0.04**
Addition and Subtraction	**1.00**	1.06	0.99	**−0.17**
NAEP	**1.29**	1.01	0.82	**−0.04**

Note. Effect size is reported as Hedges *g*. Boldface type corresponds to tested effects. Effect sizes for contrasts between FRAX vs. C and FRAX+SR-GM vs. C. These effects were not tested. Effect sizes are provided for readers' edification. Int = combined intervention conditions across conditions (without and with the self-regulated learning component); C = control group; FRAX = fractions intervention without the self-regulated component; FRAX+SR-GM = fractions intervention with self-regulated learning with growth-mindset instruction; NAEP = 13 released fraction items from the National Assessment of Educational Progress.
[a]For Ordering, pretest is comparing fractions with the *Fraction Battery-revised Comparing* (Malone & Fuchs, 2017).

conditions versus control, as tested in the statistical model. (This was also the case for each intervention condition vs. control; see Table 5's second and third columns.) The findings on NAEP, the study's transfer measure indexing generalized fraction knowledge, are especially notable, because NAEP's content is similarly distal to the intervention and control conditions.

Fractions outcomes of at-risk students who received the next-iteration intervention were statistically significantly stronger than the outcomes of comparable at-risk students in the control group.

To index achievement gaps at the end of intervention, we included a follow-up sample of not-at-risk classmates who completed a subset of fractions measures before and after intervention. At-risk intervention students' achievement gaps at the end of intervention compared favorably to those of the at-risk control group (see Supplemental Table 2). On Addition and Subtraction, the at-risk control group's posttest gap dropped but remained substantial (from 1.19 standard deviations below classmates to 0.69). On Ordering and Word Problems (indexed only at posttest in not-at-risk students), sizeable posttest gaps were also evident (0.46 standard deviations for Ordering; 0.80 standard deviations for Word Problems). Most problematic, the NAEP achievement gap grew substantially (from 0.59 standard deviations at pretest to 1.44 at posttest). Across follow-along measures, the at-risk control group's posttest fraction performance gaps averaged 0.85 standard deviations.

The picture was dramatically more positive for at-risk intervention students. The NAEP gap held steady (from 0.55 standard deviations to 0.49), with a posttest gap substantially smaller than that of the control group's NAEP gap of 1.44 standard deviations. On Word Problems, it went from 0.65 standard deviations to 0.09. On other measures, intervention students outperformed not-at-risk classmates at posttest: by 0.24 standard deviations on Ordering and by 0.31 standard deviations on Addition and Subtraction. Across follow-along measures, the achievement gap closed (M = 0.01 standard deviations).

As framed in the introduction, by demonstrating significantly stronger fractions knowledge compared to at-risk control group students, with large ESs, and by demonstrating that students complete intervention with substantially narrowed achievement gaps, the present study supports the efficacy of this next-iteration fractions intervention. More broadly, these findings suggest that students with initially large achievement gaps, including those with learning disabilities, can succeed with challenging mathematics standards.

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At the same time, readers may wonder whether this study's large ESs are attributable to alignment between the researcher-developed outcome measures and the intervention's content. In this vein, consider three points. First, the ES of 1.29 for the contrast between intervention versus control on the NAEP transfer measure (not researcher designed) was similar to ESs on proximal outcomes. Second, the study's proximal measures tap consensually valued mathematics skills, measured in similar ways across research groups and on widely used commercial tests (e.g., students order three fractions from smallest to largest on three blanks). Third, state mathematics achievement tests and widely used commercial math tests sample few fraction items, which can reduce their insensitivity to students' learning generally (not just the present study's intervention effects). This is why we incorporated a measure of released NAEP fraction items to assess transfer. It is also why the literature on fractions intervention relies heavily on experimenter-designed measures (see outcomes in Ennis and Losinski's [2019] meta-analysis).

We also note that the fractions intervention literature provides multiple examples of similarly strong effects. Ennis and Losinski (2019) reported an omnibus ES of 1.17. For explicit instruction studies, the ES was 1.25; for strategy instruction, 1.48; and for Hunt (2014), the only third-grade study in that synthesis, 2.50. Barbieri et al. (2020) found ESs of 0.85 and 1.09 for at-risk sixth graders on fraction number line and fraction concepts outcomes. (Effects on fraction calculations, which were not the focus of intervention, were smaller.) Working with a less impaired group of sixth graders (initial skill between the 15th and 37th percentiles), Jayanthi et al.'s (2020) ESs ranged from 0.66 to 1.08. The present study extends those prior studies as well as the work of the Fuchs research group at fourth grade (Fuchs et al., 2013, 2014; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2016; Malone et al., in press) by demonstrating that such effects can be achieved at third grade.

SR-GM's Added Value

The study's second purpose was to assess the added value of SR-GM in the context of fractions intervention. Based on Wang et al.'s (2019) promising results and a prior third-grade classroom study focused on math problem solving (Fuchs, Fuchs, et al., 2003), we anticipated added value for SR-GM. In contrast to our hypothesis, the present study revealed no significant effect between fractions intervention with versus without SR-GM. Some may wonder whether this is due to inadequate statistical power. However, the mean ES between these conditions was 0.01 (–0.17 to +0.10). Therefore, the lack of statistical significance between the two intervention conditions cannot be attributed to insufficient statistical power. The absence of statistical significance is instead due to the absence of a practically meaningful effect.

In contextualizing this finding in the intervention literature, where we identified no prior studies beyond Wang et al. (2019) isolating SR-GM's effect on math performance, it is instructive to consider Graham and Harris's (1989) randomized controlled trial in which students with learning disabilities were randomly assigned to receive SRSD's writing strategies with versus without SR. As in the present study, SR provided no added value over strategy instruction alone on the writing outcome. A commonality between Graham and Harris and the present study is strong academic strategy instruction as the framework into which SR (in our case, SR-GM) is embedded. A future study might explicitly test the proposition that SR-GM provides added value when contextualized in weaker academic intervention, which may be sometimes occur in practice.

Another potential explanation for SR-GM's lack of added value is the present study's group size of two per group. It is possible that SR-GM strategies are more important when students have less individual attention and more responsibility for independent self-regulation. It is also possible that although the SR-GM component did not confer added value on fractions performance, it may have afforded benefits on outcomes not measured in the present study.

Study Limitations

Before closing, we note study limitations. First, the study design did not include follow-up data collection, so sustainability of effects is unknown. Future studies should incorporate a maintenance check 1 year later to assess whether intervention effects maintain. This is important because although such intervention is necessary, given its clear superiority over the control group, the field needs to know whether such intervention is sufficient to produce adequate long-term superior outcomes. Instead, a sustained approach to intervention may be needed for at least some segment of the at-risk intervention population. Research on key predictors of follow-up success is also important to provide schools with direction for identifying students in need of sustained intervention, perhaps on the basis of a combination of variables collected at pre- and posttest. Second, readers should note that our description of the schools' classroom and intervention instruction is based solely on teacher self-report data, as are estimates of the mathematics instructional time students received.

Conclusions and Implications for Practice

With these limitations in mind, we offer the following conclusions. First, intervention that (a) builds integrated thinking across fraction magnitude activities while encouraging strategic distinctions among those activities; (b) supports recognition of problem types and accurate solutions using efficient strategies via explicit, structured instruction; (c) encourages mastery via an instructional pace that permits deep understanding and practice; (d) includes systematic interleaved practice; and (e) provides ongoing corrective feedback may reduce or eliminate the need for SR-GM instruction. Second, fractions intervention designed with these principles promotes a high level of outcomes for third-grade at-risk students on grade-level career- and college-ready standards.

Additionally, present findings should encourage school personnel to seek interventions for at-risk learners that simultaneously address grade-level content while attending to foundational skills. Such an approach may bridge the ongoing divide between general education's emphasis on challenging standards for all students and special education's individualized goal-setting framework (Edgerton et al., 2020). Finally, given present findings indicating at-risk third graders can succeed on challenging fractions, schools should consider rethinking classroom instruction (as represented in the present study) to address fraction magnitude more effectively.

Findings should encourage school personnel to seek interventions for at-risk learners, which simultaneously address grade-level content while attending to foundational skills.

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Supplemental Material

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