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Next-Generation Fraction Intervention and the Long-Term Advantage of Interleaved Instruction

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Abstract

This study's first purpose was to investigate effects of a fourth- and fifth-grade "next-generation" fraction intervention, which included six enhancements over a previously validated fraction intervention, designed to address career- and college-readiness standards. The study's second purpose was to assess effects of the next-generation fraction intervention at follow-up, 1 year after intervention ended. The third purpose was to isolate the effects of one of the six intervention enhancements: interleaved fraction calculations instruction. Students with intensive intervention needs were randomized to next-generation fraction intervention (Super Solvers [SSINT]) with blocked calculations instruction (SSINT_B), SSINT with interleaved calculations instruction (SSINT_I), and control. On a mix of proximal and transfer outcomes, SSINT (across conditions) produced strong, significant effects over control at posttest. At follow-up, effect sizes were weaker but remained significant on calculations: $g = 1.22$. On other measures, follow-up g was 0.39 to 0.58. The effect of SSINT_I over SSINT_B, although not significant at posttest ($g = 0.28$), was statistically significant and large at follow-up ($g = 0.65$), in line with the cognitive science literature showing long-term advantages for interleaved instruction. Results suggest next-generation fraction intervention efficacy for intensive-needs students and the importance of interleaved instruction.

Understanding of and procedural competence with fractions are strong predictors of algebra and other forms of more advanced mathematics learning (Booth et al., 2014; Empson & Levi, 2011). Fractions are also required in many technical fields and in many everyday life situations (Gabriel et al., 2013). Yet, this strand of the mathematics curriculum is challenging for many students (e.g., Durkin & Rittle-Johnson, 2015; Kallai & Tzelgov, 2009; Siegler et al., 2011) and especially problematic for students who struggle with whole-number learning in the primary grades. For example, Namkung et al. (2018) reported that students with below-grade-level whole-number knowledge in the primary grades are 32 times more likely to struggle with fractions than are classmates with adequate whole-number knowledge. This indicates the need for fraction intervention to

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supplement classroom instruction for these at-risk learners.

In a series of randomized controlled trials (RCTs), Fuchs and colleagues (L. Fuchs et al., 2013, 2014; L. Fuchs, Malone, et al., 2016; L. Fuchs, Schumacher, et al., 2016) tested effects of a fourth-grade intervention focused primarily on fraction magnitude understanding. Across RCTs, outcomes favored intervention over control on multiple forms of fraction knowledge for students who began fourth grade with whole-number difficulty (half the sample < 15th percentile; half between 15th and 34th). Effect sizes (ESs) ranged from $g = 0.37$ to 2.50, with most in the moderate-to-large range.

As shown in L. Fuchs et al. (2015), however, the focus in U.S. schools changed over the course of these studies due to national reform centered on career- and college-readiness standards (CCRS). These standards substantially increased the depth and challenge of the fractions curriculum at third through fifth grades. This context altered the necessary scope of fraction intervention at grades 4 and 5, requiring a "next-generation" fraction intervention.

We therefore formulated a next-generation fraction intervention at grades 4 and 5 designed to strengthen our previously validated fraction intervention (e.g., L. Fuchs et al., 2013) with six major enhancements. This next-generation fraction intervention is known as Super Solvers (L. Fuchs, Malone, et al., 2021). (For information on the third-grade next-generation Super Solvers fraction intervention, see L. Fuchs, Wang, et al., 2021.) The first three enhancements addressed the intervention's fraction magnitude component. We consolidated strategies to integrate magnitude understanding and strategy use across comparing, ordering, and number line activities to deepen student understanding that fraction magnitude is involved across activities. We also added instruction to highlight similarities and differences among the magnitude activities and strengthened reliance on interleaved magnitude problem sets. This was to provide students practice discriminating among magnitude problem types. Our fourth enhancement addressed the intervention's fraction component. We addressed all four operations and problem types in quick succession and incorporated interleaved instruction, in which problem sets incorporate all problem types from the first calculations lesson forward. This was motivated by evidence that learning improves when students are required to discriminate among the highly confusable problem types represented in fraction calculations (Braithwaite & Siegler, 2018).

The last two enhancements involved supports to promote self-regulated learning, self-monitoring and goal setting, and a growth mindset to address the challenging nature of the intervention's magnitude and calculations instruction. Growth mindset, in which individuals believe intelligence can change, predicts achievement (e.g., Blackwell et al., 2007), and some prior studies reveal positive effects of such instruction in math (De Corte et al., 2000; Wang et al., 2019; Yeager et al., 2019). We infused this content within scenarios conveyed via comics (Mitchell & Milan, 1983; Obare et al., 2013) depicting similarly aged students with similar struggles engaging in the targeted processes. This is in line with social learning theory (Bandura, 1986). With self-monitoring and goal setting, students rely on progress-monitoring feedback to formatively evaluate their own progress and set goals; this is thought to help them adjust strategies (Graham & Harris, 1997) and mobilize and sustain effort (Cervone & Wood, 1995). Thus, our sixth enhancement was to incorporate systematic progress monitoring on the intervention's full set of targeted skills via curriculum-based measurement probes.

Meanwhile, as CCRS increased the curricular depth and challenge of fraction instruction at third through fifth grades, it also accelerated fraction learning at these grades (L. Fuchs et al., 2015). This type of evolving counterfactual undermines existing estimates of fraction intervention efficacy. Lemons et al. (2014) illustrated this phenomenon by examining the pattern of effects in a series of RCTs focused on a supplemental reading intervention conducted prior to and during Reading First. The reading performance of kindergarten control groups dramatically increased with Reading First's accelerated kindergarten classroom instruction; accordingly, the added value and efficacy of the supplemental reading intervention declined. This was because control groups gradually caught up to the intervention group's reading performance. Findings suggest the need not only to update interventions during periods of education reform but also to judge interventions against the next-generation counterfactual.

This situation exists anew, with CCRS national reform's deepened curricular emphasis on fractions. Given the altered counterfactual and the enhanced intervention designed to address CCRS, this study's first purpose was to assess the validity of the next-generation intervention against the next-generation counterfactual at grades 4 and 5. Our second purpose was to assess effects 1 year after intervention ended. Our third purpose was to isolate the effects of interleaved fraction calculations instruction by including two fraction intervention conditions: one with interleaved fraction calculations instruction and the other with blocked fraction calculations instruction.

Prior Fraction Intervention Studies for Students With Math Difficulty at Grades 4 and 5

As discussed, although the L. Fuchs et al. (2017) research program reveals strong efficacy for fourth-grade fraction intervention, that line of studies was conducted before and during early implementation of CCRS reform (Edgerton & Desimone, 2018). This limits generalizations of findings to present-day students. This limitation also pertains to the studies in the most recent meta-analysis of the fraction intervention literature (Ennis & Losinski, 2019). Thus, need exists for programmatic updating and additional efficacy study at fourth grade.

Yet, at grades 4 and 5, the present study's focus, we identified only one RCT conducted with students with math difficulty and testing CCRS-aligned intervention. Jayanthi et al. (2021) reported strong overall outcomes on 186 students, approximately half of whom had been randomized to receive intervention. ESs favoring intervention over control ranged from $g = 0.66$ to 1.08, but participation was limited to students whose pretest math scores fell between the 15th and 37th percentiles. Exclusion of students below the 15th percentile precludes generalizations to the most challenged learners.

Also, neither the Jayanthi et al. (2021) fifth-grade study nor the L. Fuchs et al. (2017) fourth-grade research program assessed long-term effects. In fact, we located no study at grade 4 or 5 reporting follow-up outcomes. Two studies indexed maintenance within the same school year, 7 weeks after intervention ended, among sixth graders who met the study's risk criterion for poor math outcomes. Dyson et al. (2020) reported strong effects favoring intervention over control at posttest on number line, concepts, and computation (respectively, $g = 0.90$, 0.99, 0.69); at 7-week maintenance, ESs were 1.02, 0.63, and 0.35. With a similar sample, Barbieri et al. (2020) found comparable results on conceptual outcomes, $g = 0.82$ to 1.09 at posttest and 0.60 to 0.66 at maintenance. On computation, a minor intervention focus, effects were small and not significant, $g = 0.17$ and 0.11. Thus, as expected from primary-grade whole-number intervention follow-up research (see Powell et al., 2021, for summary), effects decreased even within a limited follow-up time frame. In contrast to most primary-grade whole-number studies (see Powell et al., 2021), maintenance ESs within a similar time frame revealed practically important advantages favoring fraction intervention over control. Research on the persistence of effects over a longer time frame and at grades 4 and 5 is needed.

Prior Research on Interleaved Instruction

The dominant approach in mathematics textbooks, school instruction, and school intervention is blocked instruction (Tian et al., 2022). With blocked calculations instruction, the teacher focuses on a single operation (or problem type within an operation), with practice involving solving problems all with the same problem type. Even when cumulative review of previously taught content is incorporated, problems of the same type are blocked (i.e., grouped together). The less common approach is interleaved instruction (Tian et al., 2022), which addresses more than one operation (or problem type) at the same time and provides practice on different problem types mixed together even before all problem types have been taught. This alerts students early on that they must distinguish among problem features to identify problem types with known solutions. Gradually, the pool of known problem types expands.

Cognitive science demonstrates that although interleaved instruction tends to produce confusion and more errors early in the learning process, outcomes favor interleaved instruction over blocked instruction when adequate time is provided for consolidation (Bjork & Bjork, 2011; Rittle-Johnson & Star, 2007). Much of the interleaved literature involves self-learning experiments, in which students view category exemplars or work independently through problem sets in blocked or interleaved format, with feedback on the accuracy of answers. In either case, students decipher patterns on their own to deduce which problem features are associated with which solution methods.

By contrast, most interventions for students with math difficulty involve structured, guided instruction, in which teachers introduce problem types by modeling solution strategies while explaining and highlighting how problem features correspond to solution strategies. Students gradually take responsibility for solving problems, as teachers provide feedback to support understanding about how problem features for the problem type determine solution strategies. Only a handful of studies have investigated whether an advantage for interleaved over blocked instruction holds in the context of guided instruction.

In the most relevant study, Zigler and Stern (2016) randomized 98 typically developing sixth graders to four sessions of blocked versus interleaved instruction on algebraic addition and multiplication. In the interleaved condition, both operations were addressed together throughout the four sessions. The blocked condition treated the operations sequentially, each for two sessions. In both conditions, guided instruction was adult led. Consistent with the self-learning literature, performance during teaching sessions favored the blocked condition; however, at the end of the experiment and beyond, learning outcomes favored the interleaved condition, with a mean ES of 1.21 (Cohen's d). This is also demonstrated in the self-learning literature, in which the advantage of interleaved instruction increases after intervention ends, as students consolidate knowledge by distinguishing among problem features to identify appropriate solution strategies. Zigler and Stern (2014) provided a dramatic example, in which the blocked condition advantaged students at posttest but interleaved instruction produced stronger outcomes in the long term.

On the basis of this literature, interleaved instruction has been recommended for students with math difficulty (e.g., Carnine, 1989; Hughes & Lee, 2019; Jordan et al., 2020) and incorporated within many validated math interventions (e.g., Barbieri et al., 2020; L. Fuchs et al., 2013; Nozari et al., 2021). Yet, as discussed, interleaved instruction is rarely incorporated within school-designed intervention, and we identified no studies isolating its effects for students with math difficulty.

The Present Study's Extensions to the Literature, Questions, and Hypotheses

The present study thus extends the literature on fraction intervention at fourth and fifth grades in four ways. First, this study's intervention design incorporates enhancements to address CCRS reform and includes a control group that reflects the rigor of present-day fraction standards at both grade levels. Second, by focusing explicitly on students performing at or below the 20th percentile, we permit generalizations to students with intensive intervention needs. Third, we extended fraction intervention follow-up research by examining effects 1 year after intervention ended and by focusing on grades 4 and 5. Fourth, we isolated the effects of interleaved instruction for students with mathematics difficulty.

Toward these ends, this RCT had three arms. The first two involved the enhanced next-generation fraction intervention (SSINT) focused on magnitude understanding and calculations. The two SSINT conditions were largely the same. The difference was that fraction calculations instruction in one condition was interleaved (SSINT_I); in the other, it was blocked (SSINT_B). The third condition was a control group representing the standard program for students with intensive intervention needs during full (i.e., more mature) implementation of CCRS reform.

We had three research questions: (a) Does SSINT produce stronger fraction magnitude understanding and calculations performance for intensive needs students when compared to the standard school program in the era of CCRS reform? (b) Do effects favoring SSINT over control persist 1 year after intervention ends? (c) Does SSINT_I provide added value over SSINT_B on fraction calculations?

With respect to the first and second questions, we hypothesized stronger performance at posttest for SSINT (combined across conditions) over control and diminished but still practically important ESs at follow-up for SSINT over control, based on findings at sixth grade for 7-week maintenance (Barbieri et al., 2020; Dyson et al., 2020). For the third question, we hypothesized significant effects favoring SSINT_I over SSINT_B only at follow-up. This is consistent with studies showing interleaved instruction's effects for typically developing learners only after the knowledge consolidation realized over time with practice in distinguishing among problem-type features and linking appropriate solutions (Rohrer, 2012). We expected that a period of knowledge consolidation is important for students with mathematics difficulty, who experience more severe challenges (Schumacher & Malone, 2017) with the highly confusable problem types constituting fraction calculations (Braithwaite & Siegler, 2018).

Method

Participants

We conducted this study in accord with our university-approved institutional review board protocol, which is charged with ensuring compliance with ethical and legal standards. To determine sample size, we conducted power analysis using the Monte Carlo facility of Mplus 7.11 (Muthén & Muthén, 1998–2013), following Muthén and Muthén (2002). The sample was drawn from a large, diverse, urban and suburban countywide school district in the southeastern United States. Participants were fourth- and fifth-grade students with intensive intervention needs, operationalized as scoring at or below the 20th percentile at the start of the school year on the math portion of Wide Range Achievement Test (4th ed.; WRAT-4; Wilkinson & Robertson, 2006). This is in line with clearinghouse guidelines, such as those used by the U.S. Department of Education's National Center on Intensive Intervention (https://intensiveintervention.org). In the study's population, this screening measure is predictive of end-of-grade conceptual and calculations fraction performance (Namkung et al., 2018).

From a randomly selected pool of 207 students who met this criterion, we excluded 33 who scored below the 9th percentile on both subtests of Wechsler Abbreviated Scales of Intelligence (2nd ed.; WASI; Wechsler, 1999), because SSINT was designed to address the needs of students whose intellectual ability falls in the broadly average range. Teachers excluded two students with very low English proficiency, two due to scheduling challenges, and five for nonspecific reasons. Six moved, and one revoked assent prior to random assignment.

The remaining 158 students were randomly assigned at the individual student level to three conditions: 51 to SSINT_I; 54 to SSINT_B; 53 to a control group (standard school practice, with fraction classroom instruction and supplemental intervention for some students). Posttest analysis was completed with 142 students: 44 in SSINT_I (four moved out of county, one had a schedule change, and two were dropped when their tutor left and schedules precluded moves to other intervention groups in the same condition); 48 in SSINT_B (three moved out of county, one had a schedule change, one was removed by the teacher, and one student revoked assent); and 50 control group students (three moved out of county). See Consort Diagram in the supplemental materials.

In the posttest sample, screening scores for the three conditions, respectively, were as follows: on WRAT-4, 81.55 (*SD* = 5.66), 82.12 (*SD* = 5.00), and 82.08 (*SD* = 5.81); on WASI Vocabulary, 46.68 (*SD* = 8.52), 45.60 (*SD* = 7.55), and 46.22 (*SD* = 8.35); and on Matrix Reasoning, 41.98 (*SD* = 9.05), 40.92 (*SD* = 7.99), and 42.56 (*SD* = 8.92). In the three respective conditions, 59%, 58%, and 46% were female. Race-ethnicity was 43%, 46%, and 42% African American; 27%, 23%, and 28% White non-Hispanic; 20%, 29%, and 24% Hispanic; and 9%, 2%, and 6% Other. The percentage receiving special education was 18, 8, and 6; for English learner services, 25, 19, and 14; and for the free or reduced lunch subsidy, 48, 54, and 56.

When follow-up testing began 1 year after intervention ended (late February 2020), 21 of the 142 posttested students had moved out of county, leaving 121 (37 SSINT_I, 41 SSINT_B, 43 control). When the school district closed on March 12 2020 due to COVID pandemic, follow-up testing had been completed with 65 students (22 SSINT_I, 22 SSINT_B, 21 control). Grades 5 and 6 schools did not reopen until February 2021, and accompanying logistical complications minimized postclosure follow-up data collection to 18 additional students in spring 2021. Given effects of the yearlong in-person closure on learning, interpretation of spring 2021 follow-up data was complicated, precluding inclusion of those 18 students in follow-up analysis.

According to the 4.1 What Works Clearinghouse (WWC; 2020) standards handbook, "losing sample members after random assignment because of acts of nature is not considered attrition when the loss is likely to affect intervention and control group members in the same manner" (p. 11). In the present study, follow-up loss was not significantly different by condition, $\chi^2(2) = 0.60$ ($p = .74$), and there were no significant pre- or posttest differences as a function of condition for students who were and were not tested at follow-up.

In the follow-up sample, screening scores for the three conditions, respectively, were as follows: on WRAT-4, 82.91 (*SD* = 5.09), 83.00 (*SD* = 3.98), 80.76 (*SD* = 6.73); on WASI Vocabulary, 46.64 (*SD* = 9.53), 44.59 (*SD* = 7.71), 46.43 (*SD* = 9.76); and on Matrix Reasoning, 42.59 (*SD* = 9.61), 40.91 (*SD* = 7.41), 42.86 (*SD* = 10.51). In the three respective conditions, 68%, 46%, 52% were female. Race-ethnicity was 50%, 50%, and 33% African American; 27%, 23%, and 38% White non-Hispanic; 9%, 27%, and 29% Hispanic; and 14%, 0%, and 0% Other. The percentage receiving special education was 18, 5, and 14; for English language services, 14, 14, and 19; and for subsidized lunch, 46, 55, and 67.

Screening Measures

With WRAT-4 Calculations (Wilkinson & Robertson, 2006), students complete 40 problems of increasing difficulty: simple to complex calculations (all four operations, whole and rational numbers), nonstandard equations, number series, rounding, and computational application problems. Sample-based α = .90. With WASI Vocabulary (Wechsler, 1999), students identify pictures (four items) and define words (38 items). With Matrix Reasoning, students select from choices to complete patterns in puzzles. Reliability at this age is .88 and .93, respectively.

Outcome Measures

Five measures represented a mix of proximal and transfer distance. The two with greater proximity to SSINT than control were Ordering Fractions and Calculations. Equivalencies was similarly proximal across conditions but was addressed with greater emphasis in control. Number Line and General Fraction Knowledge represented a transfer task across conditions.

Fraction magnitude understanding. With Ordering Fractions (12 items), students order three fractions from least to greatest; items were selected from the district's online scope-and-sequence and state standards sample units. They include a mix of fractions <1, =1, and >1; 90% have unlike numerators and denominators. See supplemental material for items. Sample-based α = .81. The pretest score used as the covariate for the ordering outcome involved comparing the magnitude of two fractions (sample-based α = .78). See supplemental material for items.

With Equivalencies (12 items), students find a missing quantity in a numerator or denominator position in two fractions. Items were selected from the district's online scope-and-sequence and state standards sample units. Finding the missing quantity involves multiplying or dividing by 2 or 3. Sample-based $\alpha = .91$.

Fraction Number Line (Siegler et al., 2011) is administered and scored via computer. Students see a number line on the screen, marked with endpoints 0 and 2 with a number at the center of the line $\left(\frac{2}{3}, \frac{7}{9}, \frac{5}{6}, \frac{1}{4}, \frac{2}{3}, \frac{1}{2}, \frac{1}{19}, \frac{3}{8}, \frac{7}{4}, \frac{3}{2}, \frac{4}{3}, \frac{7}{6}, \frac{15}{8}, 1\frac{1}{8}, 1\frac{1}{5}, 1\frac{5}{6}, 1\frac{2}{4}, 1\frac{11}{12}, \frac{5}{5}, 1\right)$. They click to estimate where the number goes (without access to paper so they cannot make marks as done with intervention strategies). Each item is scored as the absolute difference between a fraction's placement and its actual value. Absolute differences are divided by 2 (for the 0-to-2 number line) and averaged across items to yield the average absolute error. To ease interpretation, we multiplied scores by -1 (higher scores reflect greater accuracy). Test-retest reliability is .80.

Fraction calculations. With Fraction Calculations, students have 5 min to complete 12 items selected from the district's online scope-and-sequence and state standards sample units: two fraction addition (one with like and the other with unlike denominators), one subtraction (with unlike denominators), three multiplication (one with like denominators, one with unlike denominators, one with multiplication of a whole number with a fraction), and four division (one with a divisor and a dividend with the same denominator, one with both fractions with unlike denominators, one with a whole number divided by a fraction, one with a fraction divided by a whole number). Sample-based $\alpha = .89$. See supplemental material for items.

General fraction knowledge. National Assessment of Educational Progress (NAEP)–Revised includes 17 released items (U.S. Department of Education, 2000–2009). In a series of RCTs testing an earlier fraction intervention (see L. Fuchs et al., 2017), we used 22 items. In the present study, we deleted five easy part-whole understanding and five easy prealgebraic knowledge. NAEP-Revised includes 12 items from the earlier set plus two involving proportional reasoning and three identifying fractions and fraction equivalencies with pictures. Testers read each problem aloud (twice, if requested). Sample-based $\alpha = .82$. See supplemental material for items.

Commonalities Across the Two SSINT Intervention Conditions

To implement the study's intervention (L. Fuchs, Malone, et al., 2021), which is referred to as Super Solvers, obtain a manual (with all materials) at https://frg.vkcsites.org. Procedures were largely the same across SSINT conditions. In this section, we describe commonalities. The program comprises three 40-min sessions per week for 13 weeks; it was delivered in pairs within the present study, and lessons included the following activities implemented in the following order.

Brain Boost (3 min, Weeks 1–13) is designed to promote self-regulated learning and a growth mindset, based on prior studies showing positive effects on math outcomes. The content is designed to help learners understand relevance for learning about fractions: how brain power can grow with hard work, how to train the brain to boost learning, how mistakes help us learn, and the value of persevering through challenging tasks, setting SMART (specific, measurable, achievable, realistic, time-bound) goals, and directing one's own learning in light of progress. Tutors invoke and extend these ideas in other lesson activities (see supplemental material for sample comics).

Multi-Minute (5 min, Weeks 1–3; 1 min, Weeks 8–13) focuses on multiplication facts, which are needed to find fraction equivalencies and reduce fractions. Students practice skip counting Factors 2 through 8 and learn a procedure for Factor 9. Multi-Minute pauses in Weeks 4 to 7 to create time for Calculations Quest. In Week 8, Multi-Minute resumes with a 1-min practice activity in which students take turns answering questions about multiplication facts. To discourage guessing, students stop for each error to explain a strategy for finding the correct answer while time elapses. The group's goal is to beat the previous session's score. If so, each student earns one "dollar" ($1) to deposit in their "bank account" (see behavior management section later for explanation).

Fraction Action (20 min, Weeks 1–7; 10 min, Weeks 8–13) focuses on magnitude understanding. Activities include four problem types: comparing fractions, ordering fractions, placing fractions on the 0-to-1 and 0-to-2 number lines, and finding fraction equivalencies. Time on Fraction Action decreases in Week 8 to permit time for Calculations Quest.

Early Fraction Action lessons focus on the meaning of the numerator and denominator. Students learn conceptual strategies to compare fractions with the same numerator or the same denominator. For same-denominator comparisons (fractions with same-size parts), they learn to focus on the numerator to decide which has more (most) same-size parts. For same-numerator comparisons (fractions with same number of parts), they learn to focus on the denominator to decide which has bigger (biggest) parts. The program supports conceptual comparing strategies with fraction tiles and representational part-whole and number line images. As comparisons with different numerators and denominators are introduced, representations are invoked regularly.

When comparing fractions with different numerators and denominators, the strategic focus is benchmarking. Students benchmark to 1, labeling above each fraction as L1 (<1), =1, or G1 (>1). If fractions are all L1, they benchmark to 1/2, labeling below each fraction as L1/2, =1/2, or G1/2. If more than one fraction is L1/2 or G1/2, they find an equivalent fraction with the same numerator or same denominator using multiplication. Students convert G1 fractions to mixed numbers, comparing whole numbers and then labeling fractions as L1 as just outlined.

A Compare Card (see supplemental material) guides students through the strategic process for assessing relative magnitude to support integrated thinking and consistent strategies across the fraction magnitude problem types. The card is gradually faded. To gain fluency in subskills within the strategies, students practice naming fractions equivalent to 1/2 and complete two 2-min speeded games in Lessons 2 through 39. In one game, they name the bigger or say "equal" for pairs of fractions; most require benchmarking. In the other game, they assess magnitude relative to 1 and 1/2.

Calculations Quest (7 min, Weeks 4–13) addresses all four operations with like and unlike denominators and whole and mixed numbers. Instruction on addition occurs in Weeks 4 and 5, subtraction in Week 6, multiplication in Weeks 7 and 8, and division in Weeks 9 and 10. To support understanding, tutors introduce each problem type with a number line representation and a simple "go-to" problem (e.g., think 1/2 of 1/4). They use worked examples to model and explain the solution procedure, gradually transferring responsibility to students. They stress the importance of identifying the operation and problem type, before selecting the solution strategy. For addition and subtraction, problems first require 1/2 equivalencies, then non-1/2 equivalencies, in sync with Fraction Action content. Students use a Calculations Quest Card, which is gradually faded (see supplemental material; cards were specific to SSINT condition).

The final activity, Power Practice, is independent practice (i.e., tutors do not model problem solutions as students complete problems). Tutors provide corrective feedback. On 10 lessons, Super Solvers curriculum-based measurement (CBM), the progress-monitoring enhancement, replaces Power Practice. Super Challenge CBM mirrors the program's fraction magnitude content, with the same problem types assessed on each alternate form (Lessons 9, 15, 21, 27, 33, 39). Conquer Calculations CBM mirrors the program's focus on fraction calculations, with the same problem types assessed on each alternate form (Lessons 18, 24, 30, 36). Tutors connect student thinking about progress and goals to Brain Boost lessons.

The behavior management system is designed to promote perseverance through difficult tasks. Introductory lessons define rules. To monitor and provide feedback on rules, tutors set a timer at random intervals through the lesson and award $1 to each student who is following all rules at the beep. When students violate a rule, tutors provide corrective feedback and set a goal for the next interval. To promote perseverance in Power Practice, Calculations Quest, and Super Challenge, tutors preselect two problems as eligible for bonus points; they reveal bonus problems after students complete work, and each student earns $1 for each correct bonus problem. Students deposit dollars into bank accounts. At the lesson's end, students pick a reward (e.g., small toy, opportunity to help the tutor) or save dollars for higher-valued items.

Distinctions Between the Two SSINT Conditions

The difference between SSINT_I and SSINT_B centered on calculations instruction. In SSINT_I, calculation problem sets (i.e., Calculations Quest and Power Practice) presented problems with all four operations, without blocking by operation. This began in Calculations Quest's first lesson, when addition was introduced (before other operations had been taught) and continued through intervention's end. By contrast, SSINT_B problems were blocked, including only the problem type targeted for instruction that week. After all operations were introduced, each SSINT_B problem set addressed one operation, which rotated through the four operations across lessons. The SSINT_I help card consolidated the four operations (see Supplemental Figure 2), and SSINT_B provided a help card for each operation (see Supplemental Figure 3).

Tutor Training and Fidelity of Implementation

Tutors were graduate students or project coordinators, each working with two to five groups. Before intervention began, tutors participated in a 20-hr workshop, in which they read from the program manual, watched sample lessons, practiced conducting lessons with peers as pseudostudents, and received feedback. Tutors achieved 95% fidelity of implementation (FOI) before intervention began. They met weekly with project coordinators for training on the next week's content and to solve emerging issues. Project coordinators also provided weekly corrective feedback based on live observations and audio recordings.

Each intervention session was recorded. To quantify FOI, 20% of the 1,898 recordings, sampled comparably across tutors, groups, and conditions, were coded. Agreement exceeded 95% on 398 double-coded recordings. For activities common across conditions, tutors addressed 90.31% ($SD = 4.90\%$) in SSINT_B and 91.59% ($SD = 4.13\%$) in SSINT_I; for Calculations Quest, tutors addressed 90.72% ($SD = 10.67\%$) in SSINT_B and 94.11% ($SD = 6.11\%$) in SSINT_I.

School Fraction Instruction

The 32 teachers who taught math to participating students completed an instructional survey. All reported teaching fractions as part of their math curriculum; one reported not teaching fraction calculations. To guide instruction, all reported relying on the district's program adoption, Go MATH! (Houghton-Mifflin Harcourt, 2015), and the district's online scope-and-sequence and sample units for state standards. Four teachers also used EngageNY modules (https://www.engageny.org/subject/math).

For magnitude understanding, sample units addressed calculating equivalent fractions with same numerator or same denominator; using fraction models (e.g., area models, number lines), and drawing pictures. EngageNY included benchmarking and a stronger focus on number lines. For calculations, sample lessons and EngageNY addressed understanding addition and subtraction as composing and decomposing unit fractions, addition and subtraction to solve problems with unlike denominators by finding equivalent fractions, regrouping fractions greater than 1 and mixed numbers for adding and subtracting, using visual models, using word problems to model calculation problems, and checking that answers make sense.

On the survey, teachers assigned a value between 0 and 100 to indicate relative emphasis on different types of visual representations and their relative emphasis on different strategies to support magnitude understanding. See Table 1 for results, contrasted against SSINT emphases. To represent fractions, teachers relied more on pictures with shaded regions; SSINT, more on number line representations. To support fraction magnitude understanding, schools relied more on procedural strategies (i.e., finding common denominators, cross-multiplying) and part-whole thinking (i.e., drawing pictures); SSINT relied more on benchmarking, conceptual comparing (e.g., same numerator, same denominator), and number lines.

Teachers reported the order in which they introduced and reviewed each fraction calculation operation by selecting all that applied for each week. Only five teachers reported addressing all four operations for two or more consecutive weeks; four focused on all four operations only after introducing each operation one by one. Although 51% reported reviewing previously taught calculations operations, they infrequently focused on more than two operations in the same week. Thus, teachers tended to introduce and practice each operation in isolation (as in SSINT_B) rather than concurrently (as in SSINT_I).

Mathematics Instructional Time

The classroom math block averaged 75.02 min (*SD* = 44.89) per day. To receive study intervention, 27 (61%) of SSINT_I students and 29 (60%) of SSINT_B students missed 45 min of core mathematics instruction (40 min intervention; 5 min transition) or the school intervention block, noninstructional seat work, or content area instruction. Four SSINT_I, six SSINT_B, and five control students received daily school math intervention (respective minutes per day of 18.03, 15.67, and 22.50 [*SD* = 9.24, 11.67, 13.63]). Total weekly math instructional time per student averaged 397.85, 407.81, and 431.00 min (*SD* = 94.77, 103.49, 93.35), such that control students had more math instructional time than SSINT_I (ES = 0.35) or SSINT_B (ES = 0.24) students; SSINT conditions were similar (ES = 0.10). SSINT students spent a greater proportion of their math instructional time than control students in intervention (.40, .41, and .13, respectively), and some of their intervention involved SSINT, which was delivered in dyads (i.e., smaller group size than with school-provided intervention).

Table 1. Percentage of Time Spent on Varying Fraction Representations and Fraction Magnitude Strategies for School Program Versus Intervention.

Topic	Strategy or tool	School % *M*	(*SD*)	Intervention %
Representations	Fraction tiles	12.66	(8.98)	20.00
	Fraction circles	11.91	(9.94)	10.00
	Pictures with shaded regions	35.00	(15.66)	10.00
	Fraction blocks	15.51	(7.61)	0.00
	Number line	24.06	(9.46)	60.00
	Other	1.56	(6.28)	0.00
Strategies	Number lines	13.13	(8.96)	20.00
	Drawing pictures	15.31	(11.07)	0.00
	Referencing manipulatives	4.06	(5.06)	5.00
	Benchmarking fractions	14.38	(10.14)	40.00
	Defining numerator and denominator	11.56	(8.08)	25.00
	Finding common denominator	24.69	(10.47)	15.00
	Cross-multiplying	16.25	(18.79)	0.00
	Other	0.13	(3.54)	0.00

Note. For each topic, teachers allocated 100 points across the various strategies or tools listed on the survey to indicate relative emphasis each had in their instruction.

Procedure

In August through October, we screened and pretested students in one large-group and two small-group sessions (rotating on a computer to complete the number line task). We then randomly assigned students at the individual level to conditions. Research staff conducted intervention from late October through mid-February in a quiet location in the school, as done in the school's intervention. In February and March, we posttested students in one large-group and two small-group sessions. In January, teachers completed instructional surveys. Testers were trained and passed fidelity checks before screening, before pretesting, and before posttesting. To quantify FOI of test administration, 20% of audio recorded test sessions (sampled comparably across testers) were coded with an FOI checklist. FOI exceeded 97%. Research staff independently scored and entered each test twice and resolved discrepancies.

Transparency and Openness

This report describes participant exclusions, the approach used to calculate sample size, and data manipulations and analyses. This report's data are available from the first or third author; data analysis code is available from the third and fourth authors; and research materials are available from the first author. This study's design and analysis were not preregistered.

Data Analysis and Results

Preliminary analyses indicated that pretest performance did not moderate intervention effects on any fraction outcome. Multilevel analyses were conducted with Mplus 8.2 (Muthén & Muthén, 2018). Other preliminary analyses evaluated the nested structure of the data: a cross-classified, partially nested design in which nesting occurred at the school and classroom levels for all study conditions and at the intervention-dyad level for the two intervention conditions. A three-level model with cross-classification of dyad and classrooms, both nested in schools, did not converge. We therefore used an indirect strategy to estimate the proportion of variance in each fraction outcome measure due to schools, classrooms, and intervention dyads: first regressing observations on school dummy codes and then modeling student data as nested in a cross-classification of classrooms and dyads using fixed effects, controlling for schools using dummy codes. The variance components from this pair of models were used to compute intraclass correlations (ICCs; i.e., the proportion of total variance in the specified outcome attributable to the specified level). As shown in Supplemental Table 1, ICCs were large enough to justify retaining school, classroom, and dyad in analyses. Because there were only 12 schools, we used the strongly preferred fixed-effects approach, replacing a level with $k - 1$ dummy codes for cluster membership (McNeish & Stapleton, 2016). At this stage, ICC analyses indicated a Bayes estimator be used; school membership be modeled using fixed effects; and student-level outcomes be modeled as nested in a cross-classification of classroom and dyad.

We next accounted for the partial nesting of the data, in which both intervention conditions had students nested in dyads but the control condition did not. We used the Roberts and Roberts (2005) method (in Bauer et al., 2008), in which ICC for dyad was defined for SSINT_I and SSINT_B but undefined for the control group. We obtained ICC results separately for each condition, but they shared a common Level 1 residual variance.

Then we conducted regression analyses to test the contrasts of interest, using the ICC code as a basis and adding pretest scores as covariates. The contrasts of interest were intervention (SSINT; combined across conditions) versus control and SSINT_I versus SSINT_B. The final full model equation was

$$y_{ijk} = \gamma_{00} + \sum_{m=1}^{11} \gamma_{0m} d_{mjk} + u_{0j}$$

$$+(\gamma_{10} + u_{1j} + u_{1k}) c_{1ijk} + (\gamma_{20} + u_{2j} + u_{2k})$$

$$c_{2ijk} + \gamma_{30} y_{0ijk} + e_{ijk},$$

where i denotes individual student, j denotes classroom, k denotes dyad, y is a generic outcome, d is dummy code for school, y_0 is pretest, c_1 is dummy code for SSINT_B condition (control = 0, SSINT_B = 1, SSINT_I = 0), and c_2 is dummy code for SSINT_I condition (control = 0, SSINT_B = 0, SSINT_I = 1). For average (combined) intervention versus control, the difference was $[(2\gamma_{00} + \gamma_{10} + \gamma_{20}) / 2] - \gamma_{00} = 1/2(\gamma_{10} + \gamma_{20})$. For SSINT_I versus SSINT_B, the difference was $\gamma_{20} - \gamma^{10}$.

At follow-up, the nested data structure was further complicated with additional nesting at the school and classroom levels for all study conditions. To explore the effects of nesting at follow-up, we fit two sets of unconditional models. The first involved three-level models (Level 1 = student, Level 2 = follow-up classroom, Level 3 = follow-up school), in which ICCs at the school level at follow-up were minimal (less than .01), whereas classroom-level ICCs ranged from .02 to .23. Thus, we ran a second set of models where students were cross-classified into the base-year and follow-up classrooms. ICCs ranged from .06 to .41. Then we conducted a series of two-level cross-classified regression analyses to test the effects of the combined SSINT versus control and SSINT_I versus SSINT_B as fixed effects, with base-year school dummy codes at the base-year between level and pretest scores as a within-level covariate.

Table 2 shows pretest, posttest, follow-up, and adjusted means by condition (there were no missing data at pretest or posttest). Testing for equivalence revealed no significant differences among conditions on any pretest fraction measure. Results of the Bayes estimation are provided in Table 3, in which credible intervals (CrI) excluding zero indicate significant effects. (With Bayesian estimation, a 95% CrI has a 95% probability of containing the parameter. Accounting for multiple comparisons is not necessary with Bayesian analysis because it is more conservative than frequentist analysis [Gelman et al., 2012]. Also, the tests for different dependent measures are independent, and only two hypothesis tests were conducted for each outcome.)

At posttest (Table 3), SSINT (across conditions) produced stronger performance than control on all outcomes except NAEP-Revised, with SSINT_I and SSINT_B performing comparably on each outcome. At follow-up (Table 4), effects for SSINT (across conditions) over control diminished, but the effect remained statistically significant on calculations. Further, the follow-up effect favoring SSINT_I over SSINT_B was statistically significant. For ESs (adjusted posttest means divided by posttest pooled standard deviation Hedges's g; Hedges & Citkowicz, 2014), see Table 5.

Discussion

We consider findings in terms of the effects of Super Solvers intervention (SSINT; combined across conditions) compared with the control group, first on posttest outcomes (Question 1) and then at follow-up (Question 2). Then we discuss effects between interleaved versus blocked SSINT conditions at both end points (Question 3). Finally, we summarize study limitations, with implications for future research, and the study's major conclusions.

Effects of Next-Generation Fraction Intervention Over Control at Posttest

Consistent with our hypothesis, students who received SSINT (combined across conditions) experienced stronger fraction outcomes over control group students. This was the case on three of four posttest conceptual fraction outcomes. The ES on ordering was $g = 1.75$, reflecting in part this measure's closer proximity to intervention than control, than was the case for other conceptual outcomes in our battery. What also contributes to the large ES is the control group's poor showing, due to the school program's heavy instructional focus on cross-multiplication for comparing fractions (24% emphasis vs. 0% in SSINT). This "trick," which is commonly taught in schools without conceptual focus, undermines student learning because it circumvents mathematical thinking about fraction magnitude (Olanoff et al., 2014). It is also procedurally challenging when students order more than two fractions, as on the ordering outcome measure.

The meaningfulness of SSINT's advantage over control on the ordering outcome is supported by the intervention students' stronger performance on two other, important

Table 2. Pre- and Posttest Means, Standard Deviations, and Adjusted Means for the SSINT Versus Control Group Contrast and for the SSINT_I Versus SSINT_B Contrast.

	SSINT versus control										SSINT_I versus SSINT_B									
	SSINT					Control					SSINT_I					SSINT_B				
	Pretest (*n* = 92)	Posttest (*n* = 92)		Follow-up (*n* = 44)		Pretest (*n* = 50)	Posttest (*n* = 50)		Follow-up (*n* = 21)		Pretest (*n* = 44)	Posttest (*n* = 44)		Follow-up (*n* = 22)		Pretest (*n* = 48)	Posttest (*n* = 48)		Follow-up (*n* = 22)	
Measure	*M* (*SD*)	*M* (*SD*)	M_{adj}	*M* (*SD*)	M_{adj}	*M* (*SD*)	*M* (*SD*)	M_{adj}	*M* (*SD*)	M_{adj}	*M* (*SD*)	*M* (*SD*)	M_{adj}	*M* (*SD*)	M_{adj}	*M* (*SD*)	*M* (*SD*)	M_{adj}	*M* (*SD*)	M_{adj}
Ordering	3.61 (1.44)	7.50 (2.93)	7.49	5.14 (3.18)	5.08	3.56 (1.23)	2.74 (2.22)	2.76	4.00 (2.61)	4.11	3.82 (1.57)	7.25 (3.08)	7.14	5.59 (3.45)	5.25	3.42 (1.03)	7.73 (2.90)	7.83	4.68 (2.90)	4.93
Equivalencies	2.46 (2.90)	7.65 (4.29)	7.57	6.66 (4.54)	6.47	2.06 (2.72)	4.32 (4.10)	4.43	4.43 (3.70)	4.80	2.77 (3.21)	7.86 (4.12)	7.67	7.55 (4.33)	7.12	2.17 (2.57)	7.46 (4.48)	7.64	5.78 (4.67)	5.85
0–2 NL	0.54 (0.16)	0.29 (0.13)	0.28	0.34 (0.17)	0.34	0.50 (0.17)	0.45 (0.18)	0.46	0.43 (0.19)	0.43	0.53 (0.18)	0.29 (0.15)	0.29	0.33 (0.18)	0.35	0.55 (0.14)	0.28 (0.11)	0.28	0.36 (0.15)	0.34
NAEP-Revised	5.71 (3.00)	8.46 (3.61)	8.38	—	—	5.39 (2.92)	7.71 (3.86)	7.85	—	—	5.88 (3.09)	8.55 (3.59)	8.36	—	—	5.56 (2.95)	8.38 (3.65)	8.40	—	—
Calculations	1.31 (1.00)	7.55 (2.70)	7.53	5.13 (2.66)	5.15	1.23 (1.09)	2.37 (1.81)	2.41	2.57 (2.11)	2.52	1.27 (0.92)	7.92 (2.80)	7.96	6.00 (2.98)	6.05	1.34 (1.08)	7.22 (2.60)	7.18	4.25 (2.01)	4.24

Note. SSINT_I is fraction magnitude and calculations intervention with interleaved instruction; SSINT_B is fraction magnitude and calculations intervention with blocked instruction. 0–2 NL is 0–2 Fraction Number Line task (Siegler et al., 2011); lower values indicate stronger performance (closer placement to where fractions belong on the number line). Ordering is Ordering Fractions. NAEP-Revised is released fraction items from the National Assessment of Educational Progress. Calculations is Fraction Calculations.

Table 3. Posttest Results: Bayesian Estimates With Credible Intervals.

Measure	Contrast[a]	Mean difference	95% credible interval[b]	Significant	Condition with > value
Ordering	SSINT- IEA vs. SSINT_B	−0.908	[0.649, 0.649]		
	Intervention vs. control	4.883	[3.847, 5.811]	*	Intervention
Equivalencies	SSINT_I vs. SSINT_B	0.112	[−2.044, 2.283]		
	Intervention vs. control	3.199	[1.656, 4.707]	*	Intervention
0–2 NL[c]	SSINT_I vs. SSINT_B	−0.135	[−0.591, 0.899]		
	Intervention vs. control	1.821	[−2.349, 1.293]	*	Intervention
NAEP-Revised	SSINT_I vs. SSINT_B	−0.300	[−2.071, 1.581]		
	Intervention vs. control	0.506	[−0.669, 1.766]		
Calculations	SSINT_I vs. SSINT_B	0.751	[−0.463, 2.038]		
	Intervention vs. control	5.295	[4.521, 6.121]	*	Intervention

Note. SSINT_I is fraction magnitude and calculations intervention with interleaved instruction; SSINT_B is fraction magnitude and calculations intervention with blocked instruction. 0–2 NL is 0–2 Fraction Number Line task (Siegler et al., 2011). Ordering is Ordering Fractions. NAEP-Revised is released fraction items from the National Assessment of Educational Progress. Calculations is Fraction Calculations.

[a] Contrast between conditions is SSINT_I minus SSINT_B. "Intervention" refers to combined intervention conditions: SSINT_I and SSINT_B.

[b] With Bayesian estimation, a 95% credible interval has a 95% probability of containing the parameter (this is preferred to *p* values and frequentist confidence intervals).

[c] Number line values are multiplied by −1 such than higher values indicate stronger performance.

conceptual fraction outcomes. The advantage for intervention on equivalencies was large ($g = 0.74$), even though the control group allocated greater emphasis than intervention to lowest common denominators (25% vs. 15%). In terms of estimating placement of fractions on the 0-to-2 number line, stronger performance ($g = 1.20$) is likely due to the intervention's emphasis on benchmarking strategies (40% vs. 14%), which supports magnitude understanding (Reys & Yang, 1998). The study's number line task is a robust predictor of advanced math learning (e.g., Siegler et al., 2011) and an especially strong index of understanding in the present study because the intervention's paper-pencil strategies cannot be deployed on this computerized task.

Given the large performance advantages on ordering, equivalencies, and number line, the lack of significance on the fourth conceptual measure, NAEP released items (ES = 0.14), warrants attention. One might attribute the NAEP finding to CCRS's deepened fraction focus during control group instruction; however, intervention's stronger number line performance suggests otherwise. It is more likely that the NAEP result is due in part to the revised problem set's partial (25%) focus on part-whole interpretation of fractions (tiles, circles, shaded regions, blocks), which received greater emphasis in the school program (75% vs. 40%). It is also due to the revised NAEP's focus on proportional reasoning, which was not addressed in SSINT or the school program, further reducing the measure's sensitivity to differences between conditions. Idiosyncrasy in findings as a function of the ways researchers constitute NAEP problem sets indicates need for caution when interpreting effects based on this and other released NAEP

Table 4. Follow-Up Results: Bayesian Estimates With Credible Intervals.

Measure	Contrast[a]	Mean difference	95% credible interval[b]	Significant	Condition with > value
Ordering	SSINT_I vs. SSINT_B	0.707	[−1.075, 2.726]		
	Intervention vs. control	0.824	[−0.571, 2.207]		
Equivalencies	SSINT_I vs. SSINT_B	0.422	[−2.512, 3.474]		
	Intervention vs. control	0.877	[−1.54, 3.433]		
0–2 NL[c]	SSINT_I vs. SSINT_B	–0.138	[–1.51, 1.062]		
	Intervention vs. control	0.937	[–0.043, 1.861]		
Calculations	SSINT_I vs. SSINT_B	2.133	[0.249, 3.741]	*	SSINT_I
	Intervention vs. control	1.91	[0.838, 3.15]	*	Intervention

Note. SSINT_I is fraction magnitude and calculations intervention with interleaved instruction; SSINT_B is fraction magnitude and calculations intervention with blocked instruction. 0–2 NL is 0–2 Fraction Number Line task (Siegler et al., 2011). Ordering is Ordering Fractions. NAEP-Revised is released fraction items from the National Assessment of Educational Progress. Calculations is Fraction Calculations.

[a] Contrast between conditions is SSINT_I minus SSINT_B. "Intervention" refers to combined intervention conditions: SSINT_I and SSINT_B.

[b] With Bayesian estimation, a 95% credible interval has a 95% probability of containing the parameter (this is preferred to *p* values and frequentist confidence intervals).

[c] Number line values are multiplied by −1 such than higher values indicate stronger performance.

Table 5. Effect Sizes.

	Contrasts							
Measure	Intervention vs. control		SSINT_B vs. control		SSINT_I vs. control		SSINT_I vs. SSINT_B[a]	
Posttest								
Ordering	**1.77**	**(1.75)**	2.03	(2.01)	1.67	(1.65)	**–0.26**	**(–0.26)**
Equivalencies	**0.74**	**(0.73)**	0.79	(0.78)	0.79	(0.78)	**0.01**	**(0.01)**
0–2 NL	**1.20**	**(1.19)**	0.79	(0.70)	1.00	(0.99)	**0.08**	**(0.08)**
NAEP-Revised	**0.14**	**(0.14)**	0.15	(0.15)	0.14	(0.14)	**0.01**	**(0.01)**
Calculations	**2.14**	**(2.12)**	1.65	(1.64)	2.44	(2.42)	**0.28**	**(0.28)**
Follow-up								
Ordering	**0.41**	**(0.39)**	0.30	(0.27)	0.37	(0.34)	**0.10**	**(0.09)**
Equivalencies	**0.61**	**(0.58)**	0.25	(0.23)	0.57	(0.52)	**0.28**	**(0.26)**
0–2 NL	**0.49**	**(0.46)**	0.53	(0.48)	0.43	(0.39)	**0.06**	**(0.06)**
Calculations	**1.28**	**(1.22)**	0.84	(0.76)	1.36	(1.24)	**0.71**	**(0.65)**

Note. Effect sizes are Hedges's g (in parentheses corrected for small-sample bias). Bolded values correspond to tested effects. Effect sizes for contrasts between SSINT_B and control and between SSINT_I and control were not tested. We provide these effect sizes for readers' edification. SSINT_I is fraction magnitude and calculation intervention with interleaved instruction; SSINT_B is fraction magnitude and calculation intervention with blocked instruction. 0–2 NL is 0–2 Fraction Number Line task (Siegler et al., 2011). Ordering is Ordering Fractions. NAEP-Revised is released fraction items from the National Assessment of Educational Progress. Calculations is Fraction Calculations.

[a] Positive values indicate stronger performance in the interleaved condition.

problem sets. The absence of a measure with a well-motivated framework for indexing multiple forms of fraction knowledge is problematic, and future research should address this need.

As on the other three conceptual measures, intervention students performed more strongly than control students on fraction calculations ($g = 2.12$). Across all five study measures, the mean ES was $g = 1.20$. This is in line with Ennis and Losinski's (2019) meta-analytic mean ES of $g = 1.17$, based on studies comparing pre- or early-stage standards-reform era's less challenging fraction interventions against control groups that received less fraction instructional coverage and depth. In terms of prior studies conducted during full implementation of CCRS reform, we located none at fourth grade. At fifth grade, Jayanthi et al. (2021) reported g between 0.66 and 1.08, but because that study excluded students with pretest math performance below the 15th percentile, generalization to students with intensive intervention needs is not possible.

The present study thus extends the literature by demonstrating that a "next-generation" fraction intervention, including major enhancements designed to address CCRS standards, produces strong posttest effects for students with intensive intervention needs at grades 4 and 5. It is also important that the present study's counterfactual (i.e., control group) represents the enriched classroom instruction associated with CCRS national reform (L. Fuchs et al., 2015).

Effects of Next Generation Fraction Intervention Over Control at Follow-Up

The present study further extends the mathematics intervention literature by demonstrating a significant effect for this next-generation fraction intervention 1 year after intervention ends for SSINT over control. The large ES was $g = 1.22$, and 80% of SSINT students scored higher than the control group's mean 1 year after intervention ended. Effects on the other three follow-up measures, each indexing magnitude understanding, were smaller and not significant, but these ESs suggest SSINT's promise for long-term impact on other measures: On equivalencies, $g = 0.58$, with 64% of SSINT students scoring higher than the control group's mean; on number line, $g = 0.46$, with 70% of SSINT students scoring higher than the control group's mean. Nevertheless, conclusions about SSINT's long-term effects on magnitude understanding await research with larger follow-up sample size.

Two prior studies indexed maintenance in sixth graders with math difficulty in the same school year 7 weeks after intervention ended (Barbieri et al., 2020; Dyson et al., 2020). Both reported strong effects favoring intervention over control at posttest but with some sizeable decreases in ESs 7 weeks after intervention ended. Even so, the maintenance ESs revealed practically important advantages for intervention over control group students. A small follow-up literature on math interventions so far suggests that maintenance and follow-up ESs for intermediate-grade fraction interventions may be larger than for primary-grade whole-number interventions (see Powell et al., 2021 for summary), most likely due to larger intervention ESs at posttest. Even so, across the primary and intermediate grades and across whole and rational numbers, substantial decrements in ES raise concern.

In this vein, we note the possibility that persistence may be stronger for the present study's fraction intervention and other mathematics interventions if review of intervention strategies were to be provided during the subsequent school year. This might involve booster sessions, a relatively inexpensive means for potentially extending intervention's advantage over control. Future research should explore this possibility.

Additional work is also required to understand how a measure's proximity to intervention affects the indexing of persistence of effects. In the present study, fade-out was observed most strongly on ordering (persistence index = 22%), a measure mirroring a major intervention activity. The large decrease from posttest ($g = 1.75$) to follow-up ($g = 0.39$) echoes studies demonstrating that proximal outcomes are weaker predictors of long-term effects (e.g., Alvarez et al., 2022). On this basis, some clearinghouses place less value on proximal outcomes (e.g., www.evidenceforessa.org). Yet, as Clemens and Fuchs (2021) argue, proximal measures are necessary in intervention research because they reveal whether students learn what they are directly taught and thus permit insight into whether a small or an absence of effects on distal measures reflects poor transfer or an absence of learning.

The present study also underscores the possibility that proximal measures may, under some conditions, reveal meaningful sources of continuing advantage over time. This is reflected in the present study's other proximal measure: On calculations, the measure with the largest posttest ES ($g = 2.13$), follow-up effects favoring intervention over control were statistically significant with a large ES ($g = 1.22$). This is likely due to the strong relevance of the calculation outcome in the postintervention school year at grades 5 and 6, providing students opportunity to review, practice, and extend skills they learned during intervention. This, combined with the utility of fraction calculations in predicting algebra success (Barbieri et al., 2021), underscores the meaningfulness of this follow-up effect, despite its proximity to intervention content. Results therefore suggest that the value afforded by proximal measures within intervention research requires more nuance and fine-tuning than is reflected in present-day clearinghouse policies.

The Long-Term Advantage of Interleaved Fraction Calculations Instruction

Cognitive science demonstrates that although confusion and errors likely occur early into interleaved instruction, long-term outcomes favor interleaved over blocked instruction (Bjork & Bjork, 2011; Chase et al., 2010; Rittle-Johnson & Star, 2007). This was the basis for the present study's hypothesis that significant effects favoring SSINT_I over SSINT_B on calculations would be delayed until follow-up. Results support this hypothesis.

At posttest, when SSINT_I students had completed 2 weeks of independent practice with interleaved calculations problem sets, the posttest ES advantage for SSINT_I over SSINT_B was $g = 0.28$, which was not statistically significant. However, by follow-up, 1 year after intervention ended, the difference between SSINT conditions was statistically significant, and the ES of $g = 0.65$ revealed a large advantage for interleaved over blocked instruction. Further, 73% of interleaved students scored higher than the blocked condition's group mean.

In this way, the present study replicates a recurring finding in the cognitive science literature for the long-term advantage of interleaved instruction with a different population—students with intensive intervention needs—and in the context of intervention with structured, comprehensive instructional design. The conclusion is that interleaved instruction is an important component of fraction calculations intervention for this population. Future research should isolate the effects of interleaved calculation intervention related to whole numbers and algebra intervention for this study's population and for the broader spectrum of students with mathematics difficulty.

Study Limitations

Results must be considered in light of five study limitations. First, although the control group had more math instructional time than did SSINT students, the proportion of total math instructional time spent in intervention was greater for SSINT students than control group students, and some of SSINT students' intervention time was delivered in dyads (i.e., smaller group size than was the case for school-provided intervention). This study's test of intervention efficacy must, therefore, be largely understood as a contrast between next-generation SSINT intervention versus the next-generation inclusive fractions instruction.

In this vein, it is interesting to note that few fourth- and fifth-grade students with math difficulty received school math intervention, despite scoring at or below the 20th percentile in math. This is likely due to schools prioritizing reading over math instruction for students with dual difficulty (L. Fuchs et al., 2019) and prioritizing inclusion over intervention for students with learning disabilities (D. Fuchs et al., 2022). This is unfortunate because although many learners benefit from CCRS next-generation deepened and more challenging classroom fraction instruction, this is not the case for students with intensive intervention needs (L. Fuchs et al., 2015). Still, future studies should assess SSINT intervention against school-designed fraction intervention, while controlling for intervention group size.

The second limitation is this study's small follow-up sample size. According to WWC, because attrition was caused by the pandemic, a form of natural disaster, and because it applied comparably across study conditions, follow-up attrition does not compromise study quality. Yet, readers should take note that due to small sample size, the only measure on which effects achieved statistical significance at follow-up was calculations. The ESs on the other follow-up measures indicate promise for significant long-term effects on other measures; however, conclusions await research on SSINT's long-term effects with larger sample size.

Third, the intervention's behavioral management component may contribute to intervention effects. Many successful interventions at the intermediate grades for this population incorporate a motivational system because intervention requires perseverance on content with which students have a history of failure. Isolating effects of a behavioral component in the context of such interventions should be addressed in future studies. Fourth, this study's dyadic intervention delivery may be impractical in many schools. Given research showing that math instruction produces comparable results when delivered in groups of three, four, or five (Enu et al., 2015) and when delivered in groups 2 versus groups of 5 (Clarke et al., 2020), the hope is that this intervention retains efficacy in larger groups. Future research should explore this possibility. Finally, the study did not include a measure of word-problem performance, which may provide further insights into students' understanding of fraction calculations.

Main Study Conclusions

Three main study conclusions are as follows. First, when contrasted against a control group representing CCRS national reform's enriched classroom fraction instruction and students stronger fraction learning, next-generation intervention produces a strong posttest conceptual and calculation advantage for students with intensive intervention needs at grades 4 and 5. Second, 1 year after intervention ends, without any intervening attempt to sustain effects, the advantage for intervention over control decreases in strength. However, the effect on fraction calculations is significant, and ESs on conceptual measures suggest promise. Third, interleaved instruction is an important design feature within fraction calculations instruction for students with intensive intervention needs, with demonstrated long-term advantage over the same calculations instruction with blocked instruction.

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Supplemental Material

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